Open Risk White Paper

Connecting the Dots:
Economic Networks as Property Graphs

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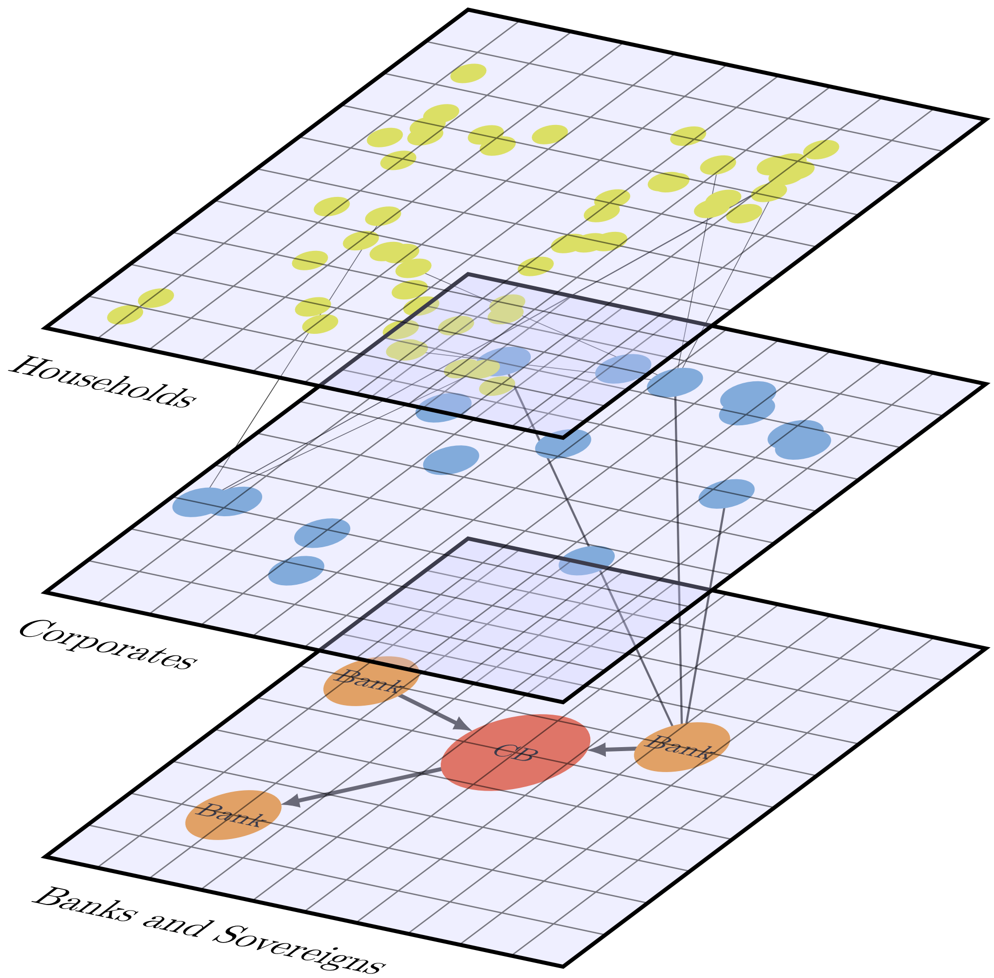

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Introduction

Economies as Networks

Interconnected agents engaging in economic activities such as trading and contracts of various forms are an essential, if not *the* defining attribute of human economies. Yet the body of classic economic literature is rarely making use of the concept of economic networks. For reasons of expediency and tractability, a number of assumptions helped abstract away the need to capture the detailed connectivity exhibited by real economies. Consequently, quantitative frameworks which describe (and potentially explain) the connectivity patterns evident in real economies have lagged other approaches. The picture is changing rapidly in recent years as many fruitful lines of inquiry are now being developed on precisely that topic.

New approaches typically make use of aspects of *graph theory* and *network analysis* to analyze economic phenomena. This is, in particular, true for networks of interconnected financial institutions, an issue that rose to prominence after the financial crisis of 2008 (e.g., [1],[2] and a recent review in [3]) but covering also broader economic aspects, e.g. core/periphery networks ([4]), the structure of multinational affiliate networks ([5]) and shadow banking ([6]).

From whom to whom data

Capturing with fidelity the internal workings of economic systems as networks helps to understand factors that may affect financial stability, such as the potential for *contagion*. A term used sometimes to denote the collection of economic network data is *from-whom-to-whom* information. In this respect, the financial crisis provided context for new thinking about how to measure vulnerabilities also in the non-financial sectors that might feed back onto the financial system, and vice versa. Empirical analysis is crucial to addressing how household and non-financial firms financial positions affect borrower delinquencies and defaults and thereby, ultimately, the balance sheet position of financial institutions. In this regard, improved data on disposable income and savings, and indebtedness of the household sector are needed. ([7], [8])

Stock-flow Consistent and Agent-Based Models

The ambition to understand the interconnected accounts between economic agents provides important motivation for collecting relevant statistics ([9]) and underpins modern flow-of-funds analysis ([10]). A body of work that revisits the basic assumptions on which economic models are being built so as to make them consistent with the flow-of-funds picture is the growing literature on *stock-flow consistent* description of economic systems ([11]). Stock-flow consistent models reflect the accounting identities that must be satisfied by the overall transaction matrix between different economic agents (See ([12])).

A related modern line of inquiry that is relevant in the economic network context are agent-based macroeconomic models. These endow the different types of heterogeneous agents with behavioral rules and allow them to interact through explicitly represented market protocols. In those models macroeconomic variables are determined by actual aggregation of the output in this population of agents[13]. Importantly, stock-flow consistent models can be augmented with the modeling of physical processes to enable insights into the ecological sustainability of economic systems ([14], [15]).

Focus on Contractual Relationships

The quantitative framework we develop here approaches economic networks from the point of view of *contractual relationships* between agents (and the interdependencies those generate). A typical use case for the proposed framework is the study of *credit networks*. The explicit modeling of credit contracts as networks, in particular involving the banking system is now a fruitful line of inquiry ([13], [16], [17]).

The main building block of network models are "nodes" - abstractions of economic agents. Nodes can own *property* (in a very generalized sense: any valuable *artefacts*) and engage also in economic exchanges and contracting of various types. Nodes are in principle any legal persons or entities (corporates, sovereigns etc). The focus is on the development of data models that document the contractual relationships between nodes and hence help clarify credit dependencies between economic agents, in particular aspects that may not be easily visible in other representations. The challenge can be seen most clearly when discussing economic networks as a collection of *balance sheets*. Widely used financial contracts, such as swaps, are difficult to represent in the balance sheet paradigm as they can be (conditional on developments) either assets with positive value or liabilities. To better represent these dependencies, financial institutions are required to make additional disclosures that are based on nominal contractual amounts ([18]). The balance sheet representation of other important contractual arrangements (labor, pension contracts, tax liabilities) is even more challenging.

Basic Concepts and Notation

In this section we start laying out the basic concepts and notation that are part of the *property graph representation*. Let us concede at the outset that a program for microeconomic description of what are factually very complex systems invariably involves many subjective choices about which features are important to highlight and capture. On the other hand a descriptive framework should be comprehensive and flexible enough so that there is no oversimplification of the networks considered.

We adopt the view that *contracting* between economic agents (and in particular credit contracts in their large variety) is the primary factor characterizing the interdependence of economic activities. This follows from the fact that the largest part of modern economic activity is based on societies where the rule-of-law is applied and seems congruent with economic scholarship ([19]). While significant economic activity is transaction based (without explicit contracting) for the core envisaged applications of this framework (credit risk) contracts play an essential role.

Money and related monetary artefacts will be treated as a distinct economic element. Money can take many different forms, in particular so in the context of modern digitally intermediated economies, in the current description only a generic representation is introduced.

The list of key concepts that underpin our idealized description of economic networks is enumerated here:[1]

1. Individual agents (both physical persons and virtual entities that have legally the same status) will be termed the nodes of the network (See Section Nodes).
2. Individual property (with its vast variety of types) that are associated with each agent (node), representing *ownership* (Assets and Services). This includes money in its various forms as an important special type of property (Subsection Money)
3. Agents engage in *exchanges* (property transfers, service provision etc.) that are abstracted as instant transactions (Section Transactions)
4. Agents enter into contracts (always assumed bilateral) that govern future transactions (Section Contracts)

Mathematically the above qualitative modeling choices will map into concrete *property graphs*. The structure of a graph, the variables that represent node states and the node interactions at any given moment and will be the way we represent the economic network.

Graph Networks

Classic Graph Networks

Graph theory is a long-standing research and application field of mathematics. Many of the core concepts and tools of graph theory have already been used in the context of financial or economic analysis.

A brief refresher: Classic non-directed graphs are an ordered pair $G = (V, E)$ comprising of:

- V a set of vertexes (also called "nodes")
- $E \subseteq \{(x, y)|(x, y) \in V^2 \wedge x \neq y\}$ a set of "edges" (also called "links"), which are unordered pairs of vertexes (i.e., an edge is associated with two distinct vertexes).

In our context, nodes will be the economic entities we want to model and edges will be the natural tool to express *relationships*. While classic graphs are already expressive and flexible tools, they have limitations that we must relax for the practical applications we envisage for this framework. In order to capture a reasonably realistic amount of information we need to expand the mathematical structure of the simple graph in various ways:

- By allowing for nodes and edges (relationships) of *different types*
- By allowing variety of qualitative and/or quantitative information to be *associated with nodes or edges*
- By allowing for a *temporal dimension*, the evolution of the graph in time.

[1]For simplicity, only a single closed economy will be considered in this framework

There is currently a growing body of work in disparate disciplines that makes use of generalized graph structures. These are going under a variety of names: *multilayer networks*, *multiplex networks* and *multidimensional graphs*. Good overviews (and links to open source software that enables working with such structures) are given in ([20],[21]). An important generalization that is suitable for our plan is the concept of a *property graph*. The term is used primarily in the context of modern graph database systems ([22], [23]). This places an emphasis on the persistent *storage* of information (versus focusing on mathematical graph properties and algorithms). This subtle shift in emphasis (representation and storage of data versus mathematical structure) is nevertheless important for practical applications. The precise research / analysis questions that users of the framework will pose are not known a-priori and are likely to evolve as more from-whom-to-whom data become available.

Property Graphs

Property graphs are both mathematical structures and realizations as concrete databases. Formally they describe *directed, labeled and attributed multi-graphs*.

- The *attributed* adjective means that both nodes and edges carry associated information (attributes or properties) that can be significantly more detailed than the simple existence of a node or a relation between nodes. The nature of those attributes (e.g. the data type) is in principle quite flexible.
- The *multi-graph* adjective means that there are multiple possible edges between nodes expressing different types of relationships. Hence the number of edges is not constrained by the product of the number of nodes.
- The *labeled* adjective means that both nodes and edges are individually identifiable and may belong to distinct types.

Property Graph Definition

A property graph is more formally defined as:

- V is a set of nodes.
- $E \subseteq V \times V$ is a set of edges.
- L_v is a set of node labels and $F_v : V \to L_v$ is the function that assigns a label to each node.
- L_e is the set of labels on entity edges and $F_e : E \to L_e$ is the function that assigns a label to each edge.
- $\Lambda_v = \{a_1, a_2, \ldots, a_n\}$ is a set of node attributes. Each node is associated with an attribute vector $[a_1, a_2, \ldots, a_n]$.
- $H_v = h_1, h_2, \ldots, h_n$ is a set of n functions such that: $h_k : V \to dom(a_k)$. For a node $v_j \in V$, the function $h_k(v_j)$ returns the value x of its k-th attribute, and $x \in dom(a_k)$.
- $\Lambda_e = b_1, b_2, \ldots, b_m$ is a set of edge attributes represented, for example, as key/value pairs. Each edge is associated with an attribute vector $[b_1, b_2, ..., b_m]$.
- $H_e = h_1, h_2, \ldots, h_m$ is a set of functions such that: $h_k : E \to dom(b_k)$. For an edge $E_{ij} \in E$, the function $h_k(E_{ij})$ returns the value x of its kth attribute, and $x \in dom(b_k)$.

Node and Edge Types and Constraints

It is important to make the presence of multiple node and edge types more explicit and represent the fact that the different types of nodes can in general interact with a subset of possible edge types.

- The set of nodes V is decomposed as $V = V^1 \cup V^2 \cdots \cup V^M$, where M is the total number of different *node types*.
- The set of edges E is decomposed $E = E_1 \cup E_2 \cdots \cup E_N$, where N is the total number of different *edge types*. Hence E_k^{mn} is the set of k-th type edges between nodes of type n and m.
- R_k^{mn} is a generalized *adjacency tensor*[2] that captures which pairs of node types can have which edges: $R_k^{mn} = 0$ indicates that for some combination of indexes (m, n) the set of k-th type edges E_k^{mn} is identically empty. In contrast $R_k^{mn} = 1$ indicates that the corresponding (m, n) node pairs may have any number of k-th type edges (including zero).

[2] In standard graphs the *adjacency matrix*, a square matrix with elements either zero or one, represents which nodes are connected

Economic Systems represented as Property Graphs

The following list provides a first sketch of the structures that we will aim to describe in more detail in (Section Nodes).

- At any given time t, the economic network is described by a set of labeled nodes V_i^c that exist at that time. Different types of nodes will express households, corporations, banks etc.
- At each timepoint every node has a list of *attributes*. Nodes have a set of properties $\{h_k(V_i)\}$ that capture, for example their ownership of any real assets. The variety of assets (tangible and intangible are to be defined in detail later (Section Assets and Services). Money (to be discussed later in Subsection Money) is an asset category of its own.
- Nodes can engage in *exchange of assets or services* via transactions. Transactions are represented as bilateral links (edges) between nodes T_{ij}. Roughly speaking, transactions express relations that are transient (not persistent in time). The order of the indexes (i, j) suggests the directionality of the relationship.
- Nodes can also engage in *bilateral contracting* with other nodes. Contracts in our context will be in general interpreted as agreements for *future transactions* (for example future exchanges of property or services at scheduled intervals). Contracts are represented as edges C_{ij} between nodes with associated labels and attributes that define the nature of the contract. Contracts (of various types, to be defined in detail later in Section Contracts).

In summary, economic agents are represented as graph nodes with various attributes, whereas their interactions are presented as edges, while we make a key distinction between present transactions and future transactions that are specified in contracts. We should note that what we capture and represent with the property graph is only the *apparent state* of nodes and their interactions.[3].

We will briefly touch on the modeling of internal node states when introducing each node type. It is important to highlight that the nodes of the economic network are (in general) not connected via any permanent (persistent in time) graph structure. Graph relations are generally formed through transactions and contracts. These have time dependent nature, hence the shape of the graph is ever mutating. There are some important exceptions to this fluid picture: A key exception comes through the fact that each node acquires a legal status (an identity or incorporation) via one or more *legal jurisdictions*. In turn such jurisdictions have a defined *geographic dimension*, which means that all nodes within a legal jurisdiction are implicitly "connected" via their common geographic association. For the same reason, these nodes a have a common and fairly persistent linkage with a *sovereign entity* or other associated "quasi-sovereign" nodes (e.g. all nodes being subject to taxation transactions) that in our graph would be represented e.g. as having tax obligations, benefiting from public services etc.

Nodes

In this section we will define more precisely the key types, properties and relationships of economic network nodes. Following loosely the Eurostat definition ([24]), a *node* is an economic and legal entity characterized by decision-making autonomy in the exercise of its principal function. To have autonomy of decision in respect of its principal function:

- An entity is entitled to *own goods and assets* in its own right. This aspect is represented with property attributes for each node which indicate what types of assets they may own.
- An entity is able to *exchange* the ownership of goods, assets or services in transactions with other units. This is represented via transaction edges that indicate the exchange of goods or the provision of services.
- Entities are able to incur liabilities, to take on other obligations or further commitments and to *enter into contracts* and accountable at law. Contracts (including those we will label as liabilities) will be represented as certain classes of edges between nodes (encoding future transactions).
- Entities are able to draw up a complete set of *accounts*, comprised of accounting records covering all the transactions carried out during the accounting period, as well as a balance sheet of assets and liabilities. In our approach the use of balance sheet accounts (with assets and liabilities) is a derived *representation* of the network and its various relationships and such representation is subject to further assumptions and valuation models. This relationship of node accounts with the property graph will be discussed more thoroughly in Section (Representations).

[3]Apparent to whom? For now we take the birds-eye view of *perfect information* that is common in economics and do not discuss alternative configurations

The list of main node types

	Real Person	Produces	Lends / Borrows	Issues Money	Issues Shares	Remarks
Households	Y	Y	L/B	N	N	Takes Employment
Corporations	N	Y	L/B	N	Y	Employs Households
Private Banks	N	N	L/B	Y	Y	
Central Bank	N	N	L/B	Y	N	
Sovereign	N	Y	B	N	N	Taxes All Nodes

Table 1: The first column indicates the legal type: All nodes except households are virtual legal constructs. All nodes engage in the consumption of goods and services of some type. Production in modern economies is concentrated in corporate nodes but this is far from exclusive: individuals and the sovereign can also produce a variety of goods and services. Banks provide important services. Labeling them as non-producing nodes simply helps highlight their more important role as *financial intermediaries*. Most nodes will engage in some type of lending (e.g savings) or borrowing contracts. Issuing money assets is the prerogative of banks (private or public / central banks). Issuing equity shares is on the other hand confined to the corporate sector and private banks

- Entities should be able to take economic decisions and engage in economic activities for which they are responsible. This is an *internal node model* requirement which we will discuss briefly but is not captured in our purely descriptive framework. The internal mechanisms for decision making and/or the detailed requirements for the production of goods and services are not modeled except implicitly via the restriction tensor R_k^{mn} (e.g. nodes that can engage in the provision of some type of service are implicitly assumed to be able to provide that service). [4]

Node Types

There is great flexibility in defining and differentiating node types in order to highlight and capture specific economic phenomena. Distinct node types can be classified along the following dimensions:[5]

1. Whether they are real persons or legal constructs. Legal entities (such companies etc.) are convenient abstractions that work as summary references to a large number of arrangements and contracts between physical persons. Some of those contracts are explicit (labor and pension contracts, management contracts and options, various financial liabilities to shareholders and lenders etc) whereas others are implicit. The difference between individuals and legal entities, while intuitively obvious, within the context of a property graph representation can only be deduced from the fact that real persons cannot engage in the same types of economic activity as other nodes (e.g. individuals do not issue equity shares, they cannot issue their own money, they cannot levy taxes etc.)

2. Whether a node can produce goods and services. In general all nodes engage in some sort of goods or services production. Production in modern market based economies is concentrated in corporate nodes but this is far from exclusive: individuals and government entities can and do produce a variety of goods and services. The corporate production profile is very wide so it has a traditional decomposition along several dozens of distinct business sectors.

3. Whether a node can create monetary artefacts (digital or physical). This is a distinguishing characteristic of bank nodes, with the distinction between private / central banks reflecting the different stakeholder base of the two.

4. The sovereign entity is very special in terms of both its connectivity (all nodes within a jurisdiction), power (its ability to enforce contracts) and distributive ability (taxation and provision of public services)

A physical person is always associated with a concrete geographical location which has been an essential aspect of economic interactions.[6]. A non-physical legal entity is not associated with a concrete location but is instead linked and inherits the locational properties of the jurisdiction that recognizes (incorporates) that legal entity. A legal entity inherits also locational properties via its interactions with physical persons (exchanges and contracts).[7]

The representative types of nodes that we will define here are shown in Table 1

An interesting practical question is whether nodes of a particular type are sufficiently homogeneous so that they can be aggregated and be represented efficiently as a single node. Conventional wisdom and practice is that household nodes

[4]Natural modeling frameworks complementing the property graph view are Agent Based Models and/or Stock-Flow Consistent Models

[5]The selection of nodes is to some degree determined by the aspects of the economy one is interested in - in this case the description of credit networks. Significant other phenomena, e.g., a pension system (if it exists), would require explicit introduction of additional financial nodes.

[6]In a digitally connected economy this may be less significant yet still a fundamental factor

[7]E.g, if a company engages exclusively in labor contracts with the citizens of a certain city and also trades within that city, it becomes economically localized as an entity within that city (even though in principle any location in its broader jurisdiction is a potential localization)

can be aggregated for various purposes whereas for corporate or private bank nodes the answer depends on their *size* (e.g. market share) which determines whether they have an idiosyncratic footprint within the network. Small and medium enterprise segments are typically aggregated, but large corporations and banks may exhibit idiosyncratic linkages that are important to capture explicitly. For many studies it is important to refine sectors and introduce *node subtypes* (e.g. the NACE classification of industrial activities), or segments of the household sector that exhibit materially different behavior. The visibility and flow of information (including both internal node *state data*, information about exchanges and contracts) is another factor that will differentiate nodes. For example, differences between private and publicly listed companies are to a good extend due the differences in the availability of information.

Internal Node States, Transitions and Models

What are these nodes really up to? The majority of their behavior is not actually motivated or explained in any detail by this essentially *descriptive* framework. Developing explanatory models for node behavior links to the work in social sciences, behavioral economics and finance. As an organizing principle we may think of our nodes as having *internal states* and exhibiting transitions on the basis of past history, external information, objectives etc. In the sequel we will survey per node type some high level aspects of internal node structure, with the understanding than explanatory models require much additional (and complex) modeling. Some relevant node behavior that *is* implicitly captured in the framework is through the asset ownership and transaction / contracting activities in which nodes are assumed to engage.

We turn next to node attributes that are explicitly modeled:

Assets and Services

Real Assets

Real assets for our purposes are abstract representations of any goods (artefacts) that can be identified as valuable, scarce, self-contained, that can be owned, transferred / exchanged, instantly consumed or kept for future use.[8]. Many assets are consumption goods (denoted C-Goods) and may be physically transformed (extinguished) in the consumption process. Many consumption assets (e.g. food) can be assumed to exist only for the time instant they are being produced for consumption (an idealization) but other assets are consumption goods that are quasi-permanent (e.g., gold or diamond jewelry).

A very important subcategory of assets has the property of *producing* - in the course of time - further assets with the addition of labor. E.g., an orchard, the machinery owned by corporation etc. Such persisting assets will in general have a longer lifespan, which may imply depreciation but not necessarily. Such long term persistent assets that have economic use are termed capital assets (C-Goods)

Many assets are transferable, i.e., they are not permanently linked to any particular set of owners, but some may be strictly linked to a person / other legal entity. Assets that are never exchanged in an economic context will not be in scope (e.g. valuable assets exchanged as gifts). Importantly, many so called *financial assets* are in our framework described more fundamentally as contracts and might only be classified as assets in the balance sheet representation we develop.

Asset Ledgers

The enumeration of relevant assets is the overall *asset inventory of the economy.* There is an index k enumerating the universe of real assets A_k. These variables may denote the quantity of asset (when quantity is a meaningful attribute of assets, e.g. one barrel of oil) or a dummy (indicator) variable when the asset is unique and indivisible (e.g. the Mona Lisa). The list of possible tradeable assets is endless. Some assets are tangible objects (e.g., a car), some are intangible (e.g., a patent). Some are quasi-permanent (e.g., land). Many produced or manufactured goods are wasting with time due to chemical processes(e.g., a cart of apples) while a few may be actually improving due to ongoing long term processes (e.g., a bottle of good wine). Some are fungible, that is, different units are practically indistinguishable and hence can be substituted for each other while others are unique and irreplaceable (e.g., the Mona Lisa or the Parthenon). Some are divisible into meaningful sub-units while others can only be considered as a whole.

[8]There are many nuances in all of these attributes. E.g ownership may mean co-ownership, what is considered *valuable* has a strong cultural element etc.

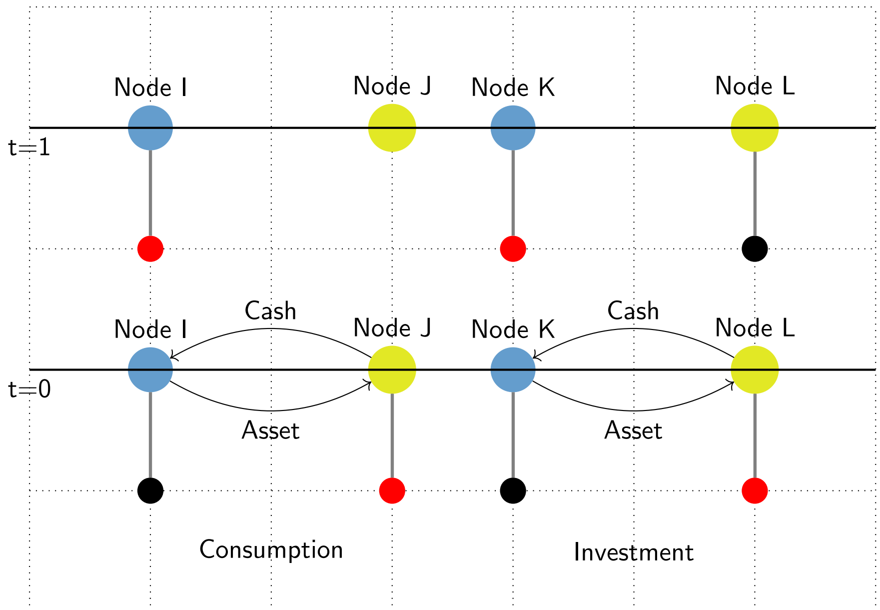

Figure 1: Visual illustration of ownership and exchange between two nodes of a consumption asset versus an investment asset. Corporate nodes are indicated as blue, household nodes as green. Cash (red) and Real assets (black) are depicted as attached (solid gray lines) to the nodes that own them. At the initial time (lower panels) the ownership of the consumption / investment assets is with the corporate node and is exchanged for cash with a household node. The exchanges are indicated by the arrows. At the next instant, the consumption asset has disappeared, whereas the investment asset has switched nodes.

Services

Services are economic exchanges that do not involve *persistent assets*. E.g a haircut, attending a lesson or visiting the doctor. They are similar to consumption assets in the sense that they lack persistence. For our purposes they can be idealized as being produced and consumed instantaneously. Hence services can be provided and consumed as part of exchanges but they are not node properties that persist over time. In this respect, service provision should be distinguished from a *service contract* that formally introduces persistence over time. A service contract will be valid over a finite time and hence can be represented as a relation between nodes during the entirety of that interval.

Money

The origins of money as a social / economic phenomenon are obscure. Karl Menger ([25]), among others, conjectured that one form of money, *commodity money*, emerges to solve logistical problems inherent in pure barter economies. The existence of pure barter economies is in dispute, as there is little empirical evidence ([19]). In any case, money is a major financial invention, a type of token asset that, by pure convention, can be exchanged into any other. Money is traditionally defined as any artefact that performs the three major functions of money, namely:

- **Medium of Exchange**. Facilitates economic and financial transactions by providing a readily exchangeable artefact. It is a stylized fact about money that trade is monetary ([26]). One side of almost all transactions is the economy's common medium of exchange. Money is (virtually) unique in every economy. Though each economy has a "money" and the "money" differs among economies, almost all the transactions in most places most of the time use a single common medium of exchange.
- **Store of Value**. It is a physically persisting artefact and it carries the (fluid) concept of *value* across different times and contexts.
- **Unit of Account** (also called numeraire). While there can be multiple units of account used for assigning a monetary value to goods and services most transactions and contracts (being future exchange transactions) are typically expressed in monetary terms (nominal values) using the same unit of account.

Modern money is sovereign issued fiat money, trading at a positive value though it conveys directly no utility or production. Money has a fixed face value by construction. The implementation of that face value can range from placing a numeral on a piece of paper to a encoding a binary number on the memory configuration of a chip card. At a deeper level, money is *debt* in the form of an *implicit contract* between the individual and society: An individual provides something of value (good or service desirable by other humans) in return for a token. He/she trusts to be able to use the token in the future to obtain something else of value from other humans. A person transacting in money has a credit vis--vis everyone and no one in particular (society owes a debt to him/her) ([27]). Money is thus a *promise to be able to pay in the future*. It requires trust among human agents; i.e. the belief by the members of a given community that money will, subsequently, be accepted by a third party in a trade.. Money in its current form (cash or electronic checking account money) is not an explicit contract. The legal status of money is captured primarily in it being *legal tender* for public and private payments (Within a jurisdiction an entity cannot refuse accepting payment in money designated as legal tender).

In the economic network representation we develop here, money is simply another node property, similar but distinct from real assets and contracts. We will not differentiate here further on the variations of money. A key distinguishing characteristic will be whether nodes owning monetary assets are free to used these for transactions unconditionally. In particular any form of money that fulfills its medium of exchange role only conditional on the state of a third node (e.g. a bank) is functionally closer to a contract. Capturing the dependencies that this introduces in an economic network would thus require an alternative model.

Transactions

Transactions (exchanges) of various forms are the hallmark of human economies. They underpin social organization such as specialization, large scale organization production and more. Transactions always involve at least two nodes. To represent transactions within the graph framework we make a key distinction between transactions that are thought of as *instant actions* versus *contracts* that extend over a finite period of time. Instant actions are idealized to happen within a discrete timepoint. Instant actions can be for example the exchange of property or services either as barter or via money.[9]. Transactions that span temporal intervals will typically involve a contract that specify some aspect of those future transactions (See Contracts).

Transfers

The simplest possible transaction is a *transfer* of an asset / or the provision of a service from entity i to entity j without a reciprocal transfer. The transfer at time t from entity i to entity j of an asset A_k is an edge T_{ij}, with the asset A_k being its only attribute (displayed in Fig. (2)).

The order of the indexes (i, j) in T_{ij} suggests the direction of the transfer. Obviously, entities i and j must be different for the transfer to make sense (no self dealing).[10]. The outcome of the transfer transaction is that the property graph gets modified (more specifically the node attributes of the participating nodes get updated).

Exchanges

An exchange is the simple combination of two transfers. A barter economy comprising exclusively of exchanges is a popular abstraction in economics textbooks as a representation of primitive economies. The only entities (nodes) required are physical persons / individuals, there is no separate unit of account or store of value, there is no contracting. All transactions are instantly cleared exchanges of real assets and/or services. It is doubtful that there ever was such a human economy ([19]) but it is a convenient way to build up the framework towards more realistic examples. An interesting isolated modern barter network example has been studied in ([28], [29]).

The only properties in barter exchanges are real assets A. The totality of exchanges at time t, between entity i and entity j of assets A_k against asset A_l is the set $\{X^t_{ij}(A_k, A_l)\}$. Since an exchange represents the quintessential economic activity it is useful to capture it as one edge representing the combined transaction (i.e., the tuple of edges (T_{ij}, T_{ji})), at time t, between entity i and entity j of assets A_k against asset A_l (as displayed in Fig. (3)). The order of both the node indexes and asset indexes is important! Also, the asset indexes must be different otherwise it implies a potentially fake exchange.

[9] Instant actions can also include any *sequences of actions* that can be assumed to happen one after the other, in close succession, without temporal delay

[10] We see here how this simple idealization does not distinguish between a large number of very different economic situations: A gift, a theft, a transfer that has an implicit understanding of some future benefit, all are transactions represented as simple transfers.

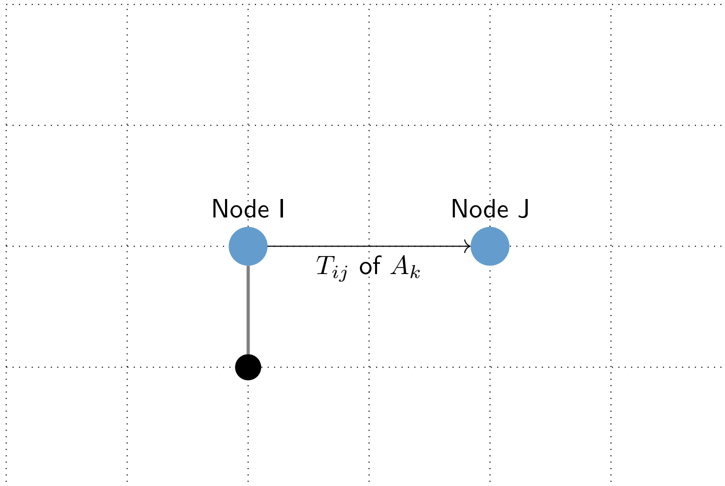

Figure 2: Visual illustration of an elementary transfer transaction. Two nodes are represented, one with an asset A_k. A directed edge between the nodes with the attribute A_k indicates that this asset is to be transfered from one node to the other

Where applicable, the *amount* of asset A_k exchanged for asset A_k is the *asset exchange rate*. In the absence of a global unit of account this rate is the only indication of the relative value of these assets to the individuals engaging in exchange.

Contracts

Contracts are legal constructs that are widely used in modern economies and govern the exchange of various assets and services. Contracts can be seen as collections (bundles) of forward transactions sets. We have already seen transactions as exchanges of cash, real assets, services, other contracts etc. Contracts are term-sheets spelling out a sequence of such future events.[11] There is an enormous variety of contracts and many are relevant in the context of capturing node dependencies in an economic network:

- Credit Contracts: Loans and other credit products which exchange money now, typically in return for nominally larger amounts in the future
- Labor / Service Contracts: Ongoing exchange of money for labor or service. In this framework labor is captured as a *pure* service by a single individual. As with services, a *labor contract* is quite distinct from the labor itself.
- Rent / Lease Contracts: This the general category of contracts which enable the use of an asset that is not owned
- Other Financial Contracts: E.g. Financial Derivatives
- Insurance Contracts

Contracts as Code

The involvement of counterparties in contracts can have multiple forms, e.g., bilateral or multilateral, securities in bearer form etc. Historically contractual details are fixed on paper, hence also the name "paper assets". While the standard manifestation of a contract is in the form of a carefully crafted legal document (occasionally running into hundreds of pages) the possibility of representing contracts using information theoretic concepts has been recognized as early as 1957 ([30]). In recent work the representation of contracts as code (automata) has been proposed. More specifically a contract can represented as finite state machine, a 5-tuple $C \equiv (Q, \Sigma, \delta, q_0, F)$. ([31])

- A finite set of states, denoted Q that describe the possible conditions of the relationship between the contracting nodes

[11] We assume here that the entire economic substance of a contact is captured explicitly. In practice such accuracy is only achievable for very restricted sets of monetary exchanges possibly contingent on carefully defined (and legally enforceable) events

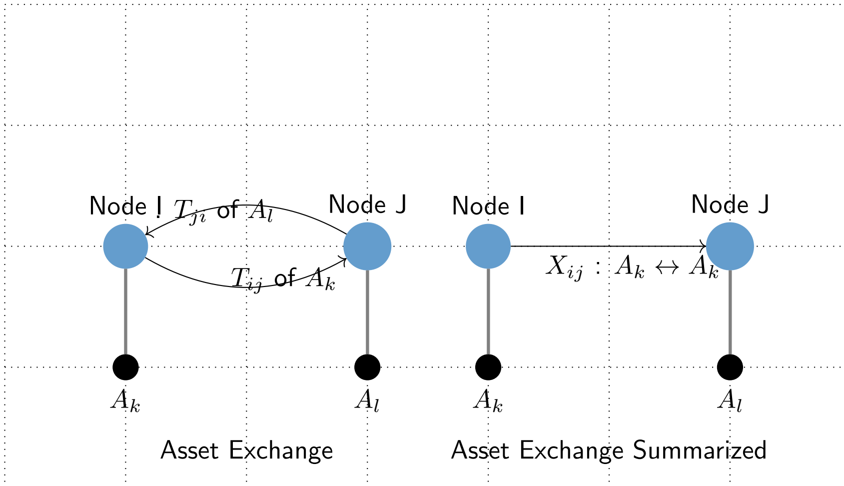

Figure 3: Visual illustration of an elementary transfer transaction. Two nodes are represented, one having ownership of an asset A_k. A directed edge between the nodes with the attribute A_k indicates that this asset is to be transfered from one node to the other

- A finite set of input symbols (events) called the alphabet (Σ). This alphabet represents the discrete set of inputs that the contract recognizes. These might correspond to systemic information events drawn from the broader economy or idiosyncratic actions taken by the contracting nodes
- The transition function ($\delta : Q \times \Sigma \rightarrow Q$ that specifies how the state of the contract changes in response to the arrival of events
- The start state ($q_0 \in Q$) (the state upon contract initiation)
- A set of end states ($F \subseteq Q$)

In theory at least, the digitalization of legal contracts allows the integration of faithful representations of contractual relationships within an information system and develop models for the evolution of the state of the system that take the encoded conditionality into account. In practice this type of representation is far from available.

Contracts as Graph Edges with Properties

Simplified representations of contracts are possible and may be adequate for various purposes. In the simplest case contract details can be captured as the numerical attributes of edges (e.g. including a list of scheduled cashflows, payment dates), with further logic of the contract encoded implicitly in the *contract type*.[12]

A contract can thus be represented as a relation between two nodes that encodes scheduled future transactions (exchanges) between the two nodes. A very important aspect of contracts is that they have duration (maturity), which may in some special cases be infinite. The contractual maturity is a future time $t_M > t$ when the final scheduled transaction *must* take place. A contract might entail a transfer from entity i to entity j of an asset A_k (can be a real asset, cash, e-money, services or anything else that is part of the economy) at contract inception $T^t_{ij}(A_k)$. In subsequent times it may stipulate transfer from entity j to entity i (hence reverse) of other assets A_l (a real asset, cash, e-money, services) $T^{t_M}_{ji}(A_l)$. The final scheduled forward transaction specified in a contract determines its *maturity*.[13]. The complete specification is that of an exchange with the various transaction legs being separated in time $C^{tt_M}_{ij}(A_k, A_l)$

Example: A loan contract

When a bank grants a loan to a borrower ([33]), both parties typically sign a contract. Ideally, it would be useful to specify in this contract all the obligations of the two parties in every possible future contingency. Even in the case of a

[12] A slightly more elaborate specification that still permits implementation in a property graph would involve including *lambda functions* as edge attributes. This is a simple means to encode conditionality (e.g. introduce payments that subject to thresholds and triggers) ([32]).

[13] The contract may indicate other legal maturity but unless it entails ongoing economic interactions is less relevant for our purposes

one-period contract, this would mean writing down a complete list of these contingencies (states of nature) at the end of the period and specifying, for each of these contingencies, the amount of the repayment to the lender. In a dynamic (multi-period) setting, things are even more complicated. A complete contingent contract would have to specify as well, in every state of nature and at every interim date:

- the amount of repayment or (possibly) the amount of additional loan,
- the interest rate on the remaining debt,
- a possible adjustment in the collateral required by the lender,
- the actions (in particular investment decisions) to be undertaken by the borrower.

Representations

While the description of node states (assets), and relationships in terms of transactions and contracts is a fairly complete set of information about an economic system, it does not lend itself to easy inspection and disclosure.

Valuation

Historically, one of the core simplification tools developed to address practical usability challenges has been the *balance sheet*. It is a stylized representation of a node's economic state at a point in time. It aims to include all material persistent properties of the node, suitably converted to an observed or postulated *monetary value*. The basic mechanism to achieve this powerful aggregation is to try convert all node property (assets) and contracts into a monetary value using the prevalent unit of account. This process is usually called *valuation*. Transactions do not feature in the balance sheet as they represent changes of state (in financial statements they might be reported in the income statement and/or the schedule of cashflows). Valuation is thus a map M from the property / contract space of a node to a numerical (monetized) value. The algorithm is as follows:

- Node properties with a non-numerical value get converted to properties with numerical value. E.g. assigning a value to the painting Mona Lisa. $M(A_k)$
- For properties that are already monetary in nature (eg cash balances) valuation is simply the identity map $A_k \to A_k$
- Edge attributes representing contracts get converted to contract values. $C \to M(C)$. For a contract, valuation captures the value of the future promised transactions it entails (both the outflows and inflows of assets)

The precise manner in which this valuation map M is performed is the art and science of *financial accounting* and it may entail widely different strategies depending on the nature of assets / contracts and the observability of their values (their exchange rates in past transactions). Generally speaking the options range depending on whether the valuation map M utilizes historical cost information about assets / contracts, in which case we denote it as a *Nominal Balance Sheet* or the current market conditions (assuming there are market infrastructures allowing the observation of appropriate values) *Market Value Based Balance Sheets*.[14]

Labeling Contracts as Liabilities

The distinction between Assets and Liabilities is in the first instance a *labeling exercise* that aims to facilitate accounting (assessing and disclosing the economic / financial state of each node). The process attaches the label 'Asset' to contracts that have positive expected monetary value and vice versa for Liabilities. As mentioned already, for a variety of important contracts this distinction is artificial and time-dependent.

We now have a detailed representation of all the tools and concepts we wanted to capture. In the next sections we will proceed to use them to describe a selection of key nodes and their relationships.

[14] For large derivative contracts subject to counterparty risk the valuation map maybe highly complex and model dependent

Household Nodes

Households are a core abstraction in the economics literature. Loosely speaking they represent the aggregate of several physical persons bound by strong biological, cultural, legal and economic ties. If persons are the atoms of the economy, households are its molecules.[15] In this work we simply assume that households are equivalent to individuals.[16].

- Households *consume* goods and services. These are modeled as exchange transactions with the corresponding goods and services producing nodes.
- Households establish labor (employment) contracts with potentially any of the other types of nodes, most importantly with the corporate sector. These labor contracts come in a large variety of forms and generate *wages* and other benefits as monetary cashflows
- Households may have a list of real asset ownership (houses, land, automobiles etc)
- Households may have cash balances (coins, bills), current accounts at commercial banks
- Holdings of Equity shares or Bonds in a variety of corporations, banks or sovereigns
- Households may borrow via Secured or Unsecured Debt Contracts, usually from banks nodes
- Households pay income taxes to the sovereign
- In principle barter between households may occur (informally)
- Traditional large households may have meaningful internal economies
- Households may engage in variety of other contracts (insurance, pensions)

Node Structure

	Symbol	**Type**	**Connection**	**Remarks**
Assets				
Real Assets	As	Asset	None	
Money	Mo	Asset	None	Cash and cash equivalents.
Transactions				
Services	Se	Transaction	Any other type of node	Individuals both consume and produce services
Goods	Go	Transaction	Goods producing nodes	Individuals mostly acquire and consume goods
Contracts				
Labor	La	Contract	Any other type of node	Labor is a contract bundling service provision (in exchange for other goods / services) over a period of time
Debt (Loans)	Lo	Contract	Bank Nodes	Large variety of debt / loans (e.g. secured / unsecured)
Term Deposits	De	Contract	Bank Nodes	
Investments	Eq, Bo	Contract	Any liability issuing nodes	Equity, Bonds etc
Tax Liabilities	Tx	Contract	Sovereign node	Taxation of household income in exchange for access to public services and goods

[15] Legally, households are less well defined than legal persons. Their existence is primarily inferred by the legally established relationships of kinship, marriage and cohabitation. There is no such thing as a "Family Legal Entity Identifier"

[16] The simplification is not without ramifications in assessing risk across an economic system. For example, married individuals may share a dwelling that is financed via a jointly repaid mortgage. If cultural norms or other circumstances change and the divorce rate changes substantially this may have implications for the sustainability of the mortgage sector

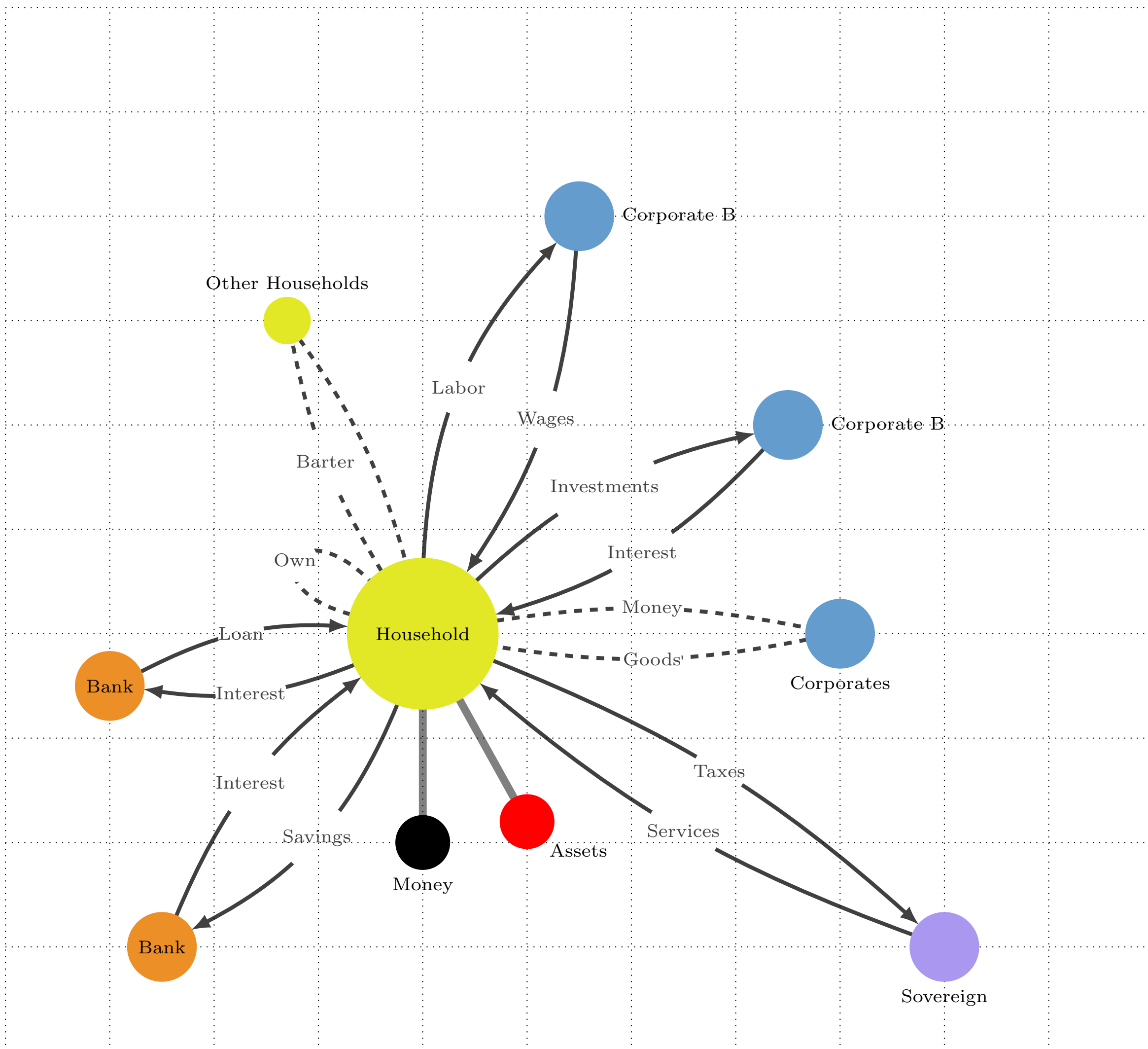

Figure 4: Visual illustration a household node and its interactions with other nodes. Various nodes and relation types as per the node structure table are depicted. Only one node / relation type is depicted, in reality a household interacts with many nodes, in particular in connection with the acquisition of goods and services

Node Balance Sheet

The elements of the stylized household balance sheet that is derived from applying a valuation function to the node's assets and contracts would be as follows:

- Real household assets (land etc.) As, with value **As** established in various asset markets (or at cost).
- Household cash balances with value **Mo**. This is simply the face value.
- Labor contracts La, with value **La**. The valuation of a labor contract can be understood as the value of the swap: a schedule of future service provision that creates value to the employer, against a remuneration scheme for the employee. A fair contract at inception of employment would have zero value and could be evolve into an asset or liability depending on the balance of value generated versus remuneration.[17]. For simplicity we focus on labor contracts with the corporate sector only.
- Household Debt Lo, with value **Lo** with value established in credit markets and reflecting the credit risk of the household
- Deposits at commercial banks, De, with value **De**. This would be close to the face value for a risk-free bank.
- Securities contracts of various types Eq, Bo, with values **Eq**, **Bo** with values established in securities markets.
- Tax Liabilities Tx with a value of **Tx**. Similarly to Labor contracts, tax liabilities are included in financial statements in idiosyncratic ways
- The net worth of the household is simply the summation of all the valuation items in scope.

Assets	Liabilities
Real Assets **As**	Household Debt **Lo**
Labor Contracts **La**	Tax Liabilities **Tx**
Cash **Mo**	
Deposits **De**	
Equity Shares **Eq**	
Bonds **Bo**	Net Worth **NW**

Corporate Entities

The traditional (industrial age) abstraction for the corporate entity is that of a production engine that involves a matrix of inputs and produces a range of outputs. Inputs are capital goods and services (labor / other) and outputs are similarly a range of capital or consumption goods or services. The corporate node category includes in-principle entities of all sizes (from mom-and-pop neighborhood stores to global multinational conglomerates). A key classification dimension is simply the number of individuals employed. Corporate nodes can also be classified according to the sector they belong (e.g. the NACE classification scheme) which determines the nature of their output (consumption or capital goods: C-Goods, K-Goods).

Complex production process means that the representation of the internal workings of corporations is introducing many extra layers of complexity (e.g. a Leontief input-output model) as captured quantitatively in ([34]). This is further complicated by the fact that large corporations do not pursue monoline business models but have conglomerate structure with countless options for selecting projects to pursue etc. The internal management of a large corporation can also involve complex remuneration schemes and options (contracts). Depending on the questions one wants to answer with the framework, certain features of the corporate structure may completely dominate others (E.g., pension liabilities could be the dominant issue over longer horizons).

- Corporates employ Households (receiving services towards production and paying wages in exchange)
- Corporates sell goods and services to most other nodes (via transactions), from households to the sovereign

[17] NB: labor relations are accounted for in corporate balance sheets only as incurred but not paid labor costs and are listed as a liability

- Corporates hold real assets (inventory) both for production purposes and potentially as investment
- Corporates hold cash (monetary assets)
- Corporates issue a variety Short / Long Term Debt Bonds and Shares (Equity), all being contracts with counterparty nodes (households, banks, other corporates etc)
- Corporates may borrow bilaterally from Banks and pay interest during the lifetime of the contract
- May invest in equities or bonds of other corporates or sovereigns for income purposes
- May invest in equities of other corporates for strategic reasons
- Pay sales tax to the sovereign
- Engage in other types of contracts (insurance, pensions)

Node Structure

	Symbol	**Type**	**Connection**	**Remarks**
Assets				
Real Assets	As	Asset	None	Both inventory and investments
Money	Mo	Asset	None	Cash and cash equivalents.
Transactions				
Services	Se	Transaction	Any other type of node	Corporates both consume and produce services
Goods	Go	Transaction	Goods producing nodes	Corporates both consume and produce goods
Contracts				
Labor	La	Contract	Household nodes	
Equity	Eq	Contract	Any node able to invest	
Debt (Loans)	Lo	Contract	Bank Nodes, Corporate Nodes	Trade credit links corporates to corporates
Term Deposits	De	Contract	Bank Nodes	
Investments	Eq, Bo	Contract	Any liability issuing nodes	Equity, Bonds etc
Tax Liabilities	Tx	Contract	Sovereign node	Sales Tax

Node Balance Sheet

Corporate Balance Sheets are of course in modern usage fairly sophisticated affairs given their use in financial disclosures (for publicly listed firms). Our stylized representation does not aim to match the standard financial statements but to highlight how the conceptual network of relationships of the corporation can be mapped into a balance sheet framework.

- Real corporate assets As, with value $\mathbf{As}$, valued with reference to a variety of markets, some more liquid than others
- Corporate cash balances with value $\mathbf{Mo}$ (face value)
- Deposits at commercial banks, De, with value $\mathbf{De}$, discounted if bank has material credit risk
- Labor contracts La with the household sector, with value $-\mathbf{La}$
- Issued Equity with value $-\mathbf{Eq}$
- Issued Bonds with value $\mathbf{Bo}$
- Investments in Equities of other corporates $\mathbf{Eq_c}$ and Bonds of other corporates and sovereigns $\mathbf{Bo_c}, \mathbf{Bo_s}$

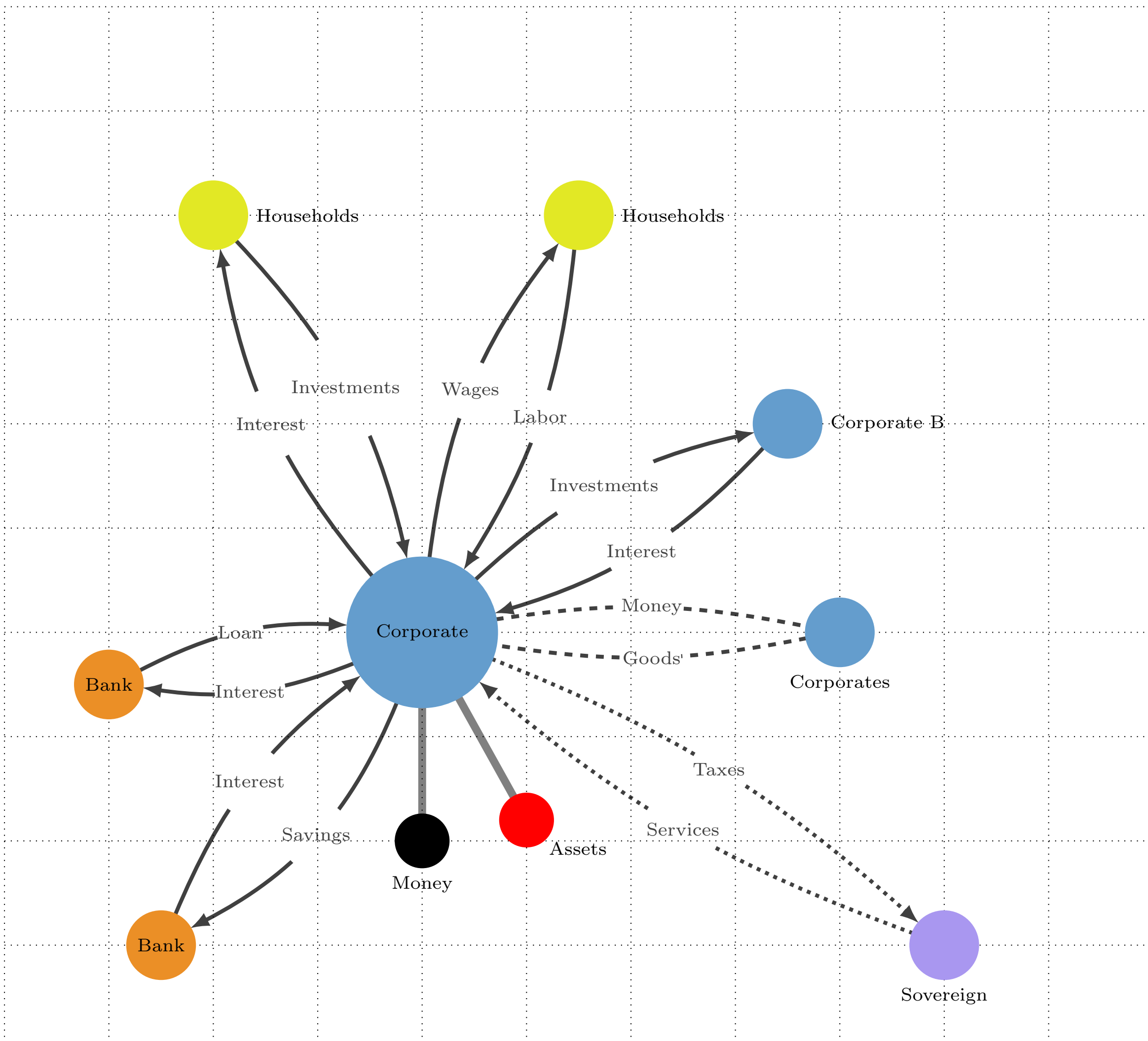

Figure 5: Visual illustration a corporate node and its interactions with other nodes. Many of the relationships we have already seen for household nodes exist here as well. A prominent new relationship represents that fact that corporates, in contrast to households are issuing a large number of securities (which households and other nodes hold as investments)

- Tax Liabilities Tx with a value of $\mathbf{Tx}$.
- The net worth of the corporate is simply the summation of all the valuation items in scope.

Assets	Liabilities
Real Assets $\mathbf{As}$	Bank Loans $\mathbf{Lo}$
Deposits $\mathbf{De}$	Labor Contracts $\mathbf{La}$
Cash $\mathbf{Mo}$	Issued Bonds $\mathbf{Bo}$
Equity Investments $\mathbf{Eq_c}$	Issued Equity Shares $\mathbf{Eq}$
Bond Investments $\mathbf{Bo_c}, \mathbf{Bo_s}$	Tax Liabilities $\mathbf{Tx}$
	Net Worth $\mathbf{NW}$

Bank Nodes

Node Structure

The internal structure of a modern bank size is quite complicated, not least because it usually involves multiple and unrelated business models. The typical (large) bank might engage in a number of distinct activities such as ([35]):

1. As Payments Provider
2. As Fractional Reserve Bank (Offering deposits, Maturity transformation (ALM and Liquidity management)) ([36])
3. As Credit Insurer (Credit underwriting (lending) including risk assessment, credit portfolio management)
4. As Capital Markets Intermediary (Issuance of Equity / Debt)
5. Performing Advisory Activities, Asset Management, Custody and other financial system related functions

Of those business lines, it is activities 2 and 3 that involve large scale bilateral contracting (and associated risk capital) and those will be the focus of the representation. Fractional reserve banking is also termed a two-tier banking system. Private bank nodes are inextricably linked to the central bank.

	Symbol	Type	Connection	Remarks
Assets				
Real Assets	As	Asset	None	
Money	Mo	Asset	None	Cash and cash equivalents
Transactions				
Services	Se	Transaction	Any other type of node	Banks both consume and produce services
Goods	Go	Transaction	Goods producing nodes	Banks consume goods such as IT infrastructure
Contracts				
Labor	La	Contract	Household nodes	
Equity	Eq	Contract	Any node able to invest in bank equity	
Loans / Other Credit	Lo	Contract	Any node borrowing from commercial banks	
Swaps and other derivatives	Sw	Contract	Other bank and corporate nodes	
Own Debt	Lo, Bo	Contract	Any node able to invest in bank liabilities	
Term Deposits	De	Contract	Other Bank Nodes	
Investments	Eq, Bo	Contract	Any liability issuing nodes	Equity, Bonds etc
Tax Liabilities	Tx	Contract	Sovereign node	

Node Balance Sheet

- Real bank assets As, with value $\mathbf{As}$ (e.g. bank branches)
- Cash balances with value $\mathbf{Mo}$ (face value)
- Reserves with the central bank $\mathbf{Re}$ (face value)
- Central Bank Credit Facilities $\mathbf{Cf}$ (face value)
- Term Deposits at other banks, De_b, with value $\mathbf{De_b}$, discounted if said banks have material credit risk
- Term Deposits from clients, De, with value $-\mathbf{De}$
- Labor contracts La with the household sector, with value $-\mathbf{La}$
- Swap contracts Sw with the corporate and bank sector, with value $\mathbf{Sw}$ (can be positive or negative)
- Issued Equity with value $-\mathbf{Eq}$
- Issued Bonds with value $-\mathbf{Bo}$
- Loans Lo to the household, corporate sector, with value $\mathbf{Lo}$ reflecting credit risk
- Investments in Equities of other corporates $\mathbf{Eq_c}$ and Bonds of other corporates and sovereigns $\mathbf{Bo_c}, \mathbf{Bo_s}$
- Tax Liabilities Tx with a value of $\mathbf{Tx}$.
- The net worth of the bank is simply the summation of all the valuation items in scope.

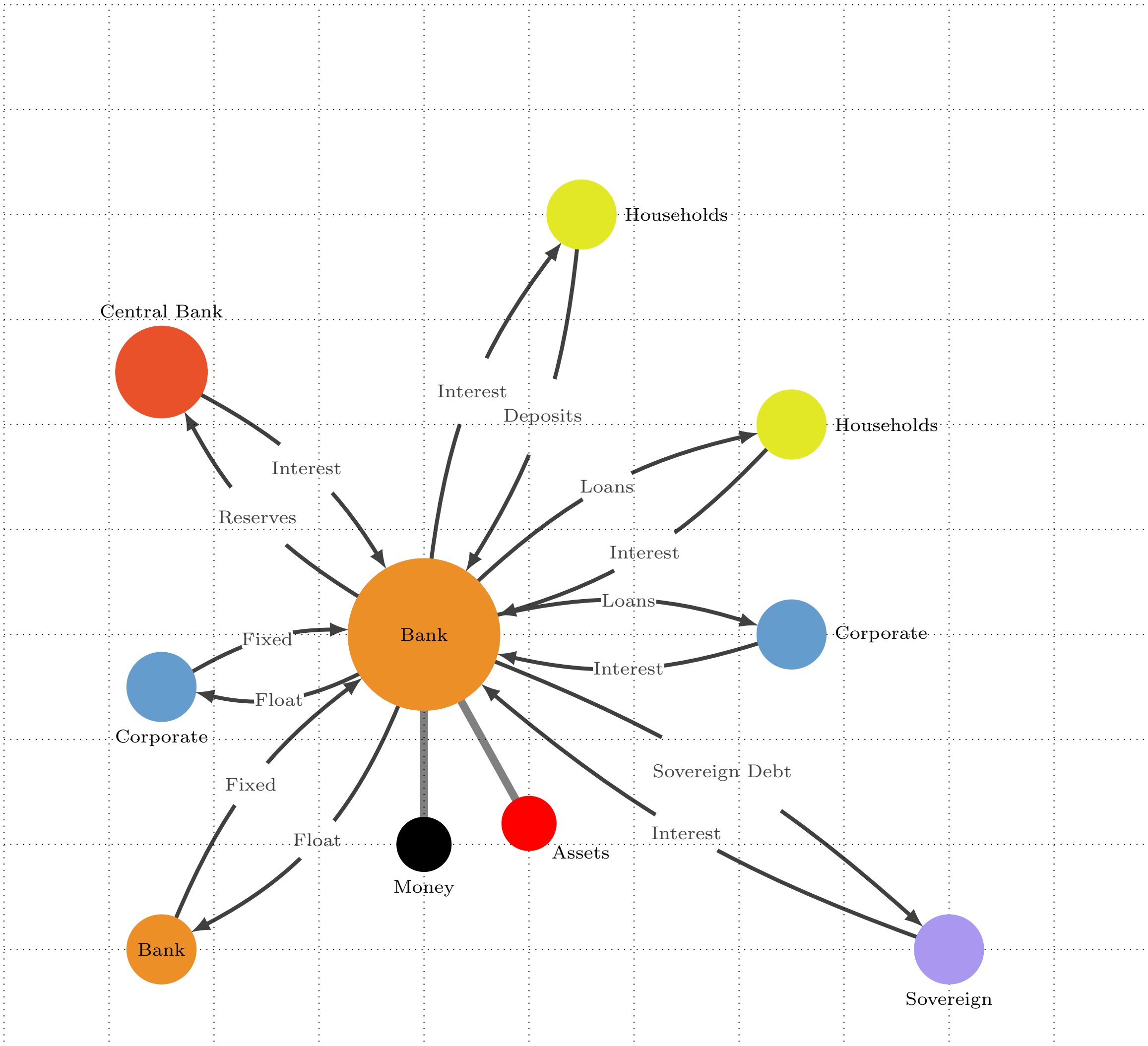

Figure 6: Visual illustration a bank node and its interactions with other nodes. First, let us note that an important new node has made its appearance: The Central Bank node. In the current two-tier banking systems only private banks and the sovereign interact with the central bank. Some existing relationships have been removed for legibility, e.g labor contracts and taxation. Banks also engage in a variety of derivatives contracts both with other banks and the corporate sector which are presented as idealized fixed-versus-floating payment legs

Assets	Liabilities
Real Assets **As**	
Cash **Mo**	
CB Reserves **Re**	CB Credit Facilities **Cf**
Deposits with other Banks $\mathbf{De_b}$	Labor Contracts **La**
Loans to Households and Corporates **Lo**	Household and Corporate Deposits $-$**De**
Derivatives **Sw**	Issued Bonds **Bo**
Equity Investments $\mathbf{Eq_c}$	Issued Equity Shares **Eq**
Bond Investments $\mathbf{Bo_c}, \mathbf{Bo_s}$	Tax Liabilities **Tx**
	Net Worth **NW**

Central Bank Nodes

In modern arrangements Central Banks are closely related but independent from the Sovereign. Central bank money is powering the two-tier banking system and is the only acceptable means to pay taxes to the sovereign. Central Bank Functions include Monetary policy. Setting the interest rate and Prudential policy. Capital / Liquidity Requirements for private bank nodes. A Central Bank is technically positioned to be bankruptcy remote. The central bank performs the following key functions:

- The central bank issues coins and banknotes for circulation in the economy (to be used as legal tender). Banknotes are neither an scarce asset for the central bank (they can be printed at will), nor a contract (banknote holders of fiat money cannot claim anything from the central bank). They are most closely represented as an artefact used in transactions.
- The central bank requires reserves from private banks on which it pays interest
- Provides private banks with credit (against collateral)
- Optionally purchases sovereign bonds or other assets
- Adjusts the configuration of the banking system via the adjustment of parameters such as interest rate, liquidity ratios and capital requirements

Node Structure

	Symbol	**Type**	**Connection**	**Remarks**
Contracts				
Banknotes	Mo	Transaction	None	
Bank Credit	Cf	Contract	Bank Nodes	
Bank Reserves	Re	Contract	Bank Nodes	
Investments	Bo	Contract	Liability issuing nodes (Sovereign)	Bonds etc

Node Balance Sheet

- Cash in circulation with value **Mo** (face value). Cash in circulation is customarily designated as a liability of the central bank.
- Reserves of private banks $-$**Re** (face value)
- Credit Facilities of private banks **Cf**

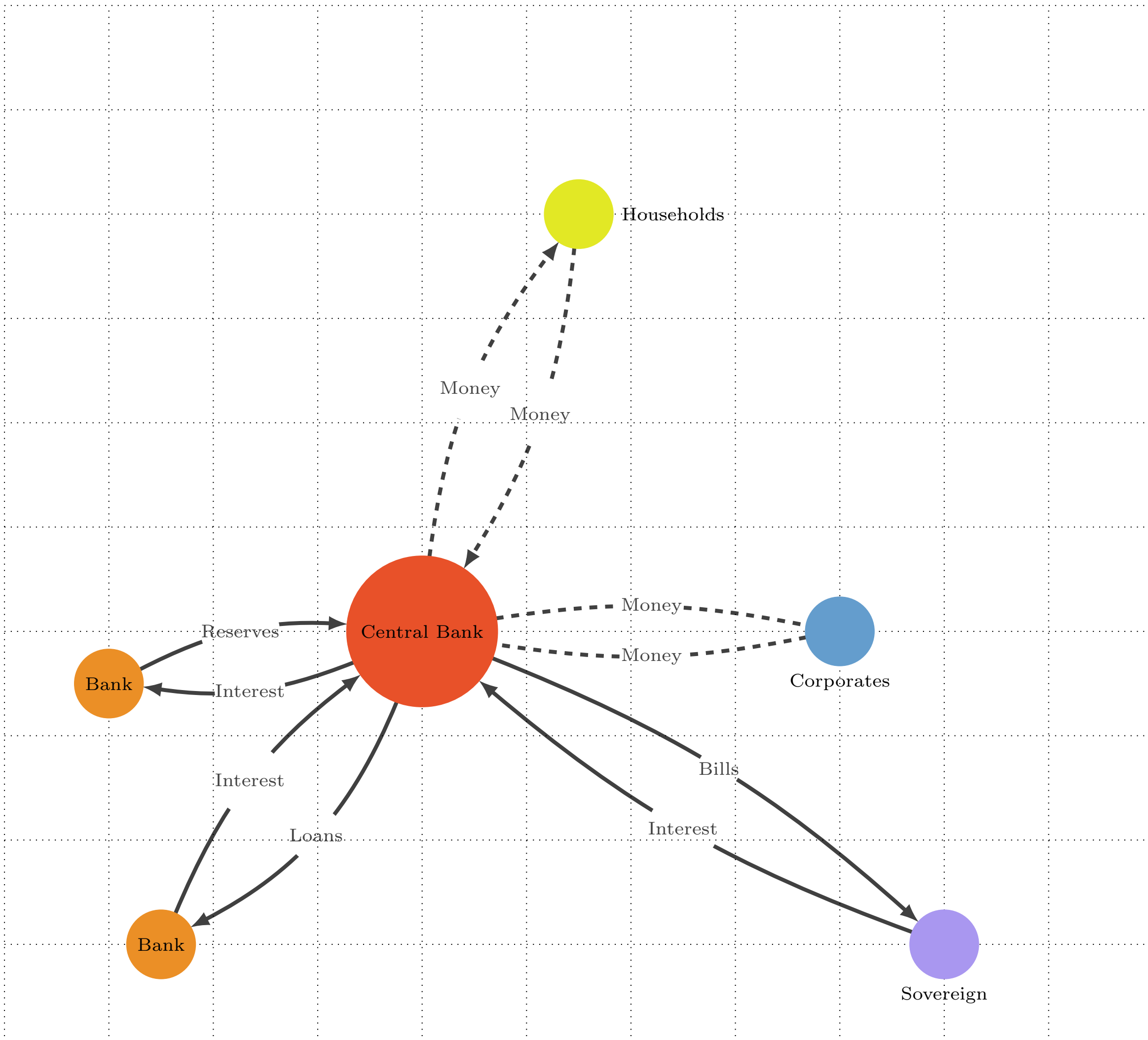

Figure 7: Visual illustration a central bank node and its interactions with other nodes. The central bank is a special node in many respects. It is unique (like the sovereign). It normally does not have relationships (e.g. claims) with entities other than private banks and the sovereign but this is a matter of policy

- Investments Bonds of the sovereign $\mathbf{Bo_s}$ (adjusted for credit risk)

Assets	Liabilities
Bank Credit Facilities **Cf** Sovereign Bond Investments $\mathbf{Bo_s}$	Cash in Circulation $-$**Mo** Reserves $-$**Re** Net Worth **NW**

Sovereign Entities

Like the central bank, a sovereign node is unlike all others. It interacts in material ways with the entirety of the economy (e.g through taxation and the provision of public services). For our purposes we summarize the sovereign entity functions as follows:

- Taxing economic activity
- Issuing government debt at different maturities
- Spending tax income on public sector projects

Node Structure

	Symbol	**Type**	**Connection**	**Remarks**
Assets				
Real Assets	As	Asset	None	A wide range of assets such as public land, mineral resources, bandwidth etc
Transactions				
Services	Se	Transaction	Any other type of node	Sovereigns both consume and produce services
Goods	Go	Transaction	Goods producing nodes	Sovereigns both consume and produce goods
Contracts				
Labor	La	Contract	Household nodes	
Sovereign Debt	Bo	Contract	Any node able to invest	
Taxes	Tx	Transaction	All economically active nodes	Sovereigns raised taxes on all economic activity

Node Balance Sheet

- Sovereign assets As, with value **As** (for many assets there might not be any real market)
- Issued Sovereign Bonds with value $-$**Bo**
- Labor contracts La with the household sector, with value $-$**La**
- Tax Assets Tx with a value of **Tx**.
- The net worth of the sovereign is simply the summation of all the valuation items in scope.

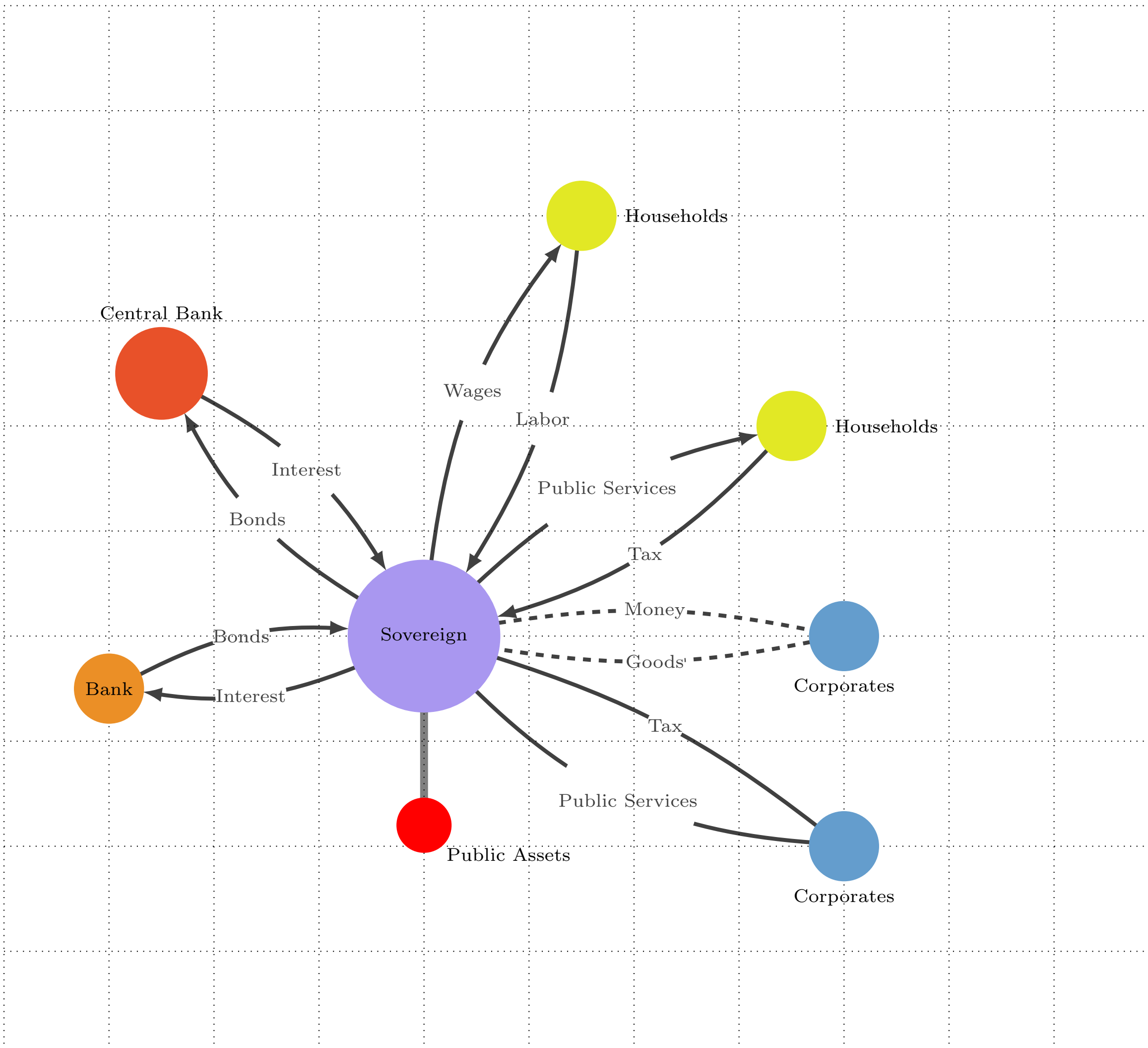

Figure 8: Visual illustration a sovereign node and its interactions with other nodes. The sovereign node interactions with all economic nodes via taxation and provision of public goods and services. It may employ a sizable fraction of households and purchase goods and service from the corporate sector. Sovereign debt is held by both the central bank and private banks

Assets	Liabilities
Real Assets **As**	
CB Reserves **Re**	CB Credit Facilities **Cf**
Deposits with other Banks $\mathbf{De_b}$	Labor Contracts **La**
Loans to Households and Corporates **Lo**	Household and Corporate Deposits −**De**
Derivatives **Sw**	Issued Bonds −**Bo**
	Net Worth **NW**

Network Representations

The Adjacency Tensor

Relationship List

Let us now collect all relationships (transactions or contracts).[18]

Name	**Symbol**	**Type**	**Description**
Services	Se	Transaction	Provision of a non-labor service
Goods	Go	Transaction	Production of consumption or investment good
Labor	La	Contract	A labor contract bundling service provision
Loans	Lo	Contract	Large variety of debt (e.g. secured / unsecured)
Deposits	De	Contract	Term deposits
Equity	Eq	Contract	Equity (shareholding) contract
Bonds	Bo	Contract	Debt in the form of security
Taxation	Tx	Contract	Taxation of income or transactions
Derivatives	Sw	Contract	Flexible contingent claim contracts

Node Relationship Matrix

Using the relationship abbreviations we can describe the adjacency of the property graph in a succinct way.

From \ To	Households	Corporates	Banks	Central Bank	Sovereign
Households	Se, Go, La	La	La, De		La, Tx
Corporates	Se, Go, Eq, Bo	Se, Go, Eq, Bo, Sw	De, Eq, Bo		Bo, Tx
Banks	Lo	Lo	Lo, De	Lo	Bo, Tx
Central Bank			De		
Sovereign	Se	Se	Bo	Bo	

The Network Balance Sheet

Putting together aggregations of each node category balance sheets (made possible through the power of conversion of all entries to monetary values) we obtain a *network balance sheet*. Compare for example with the stock-flow model INSOUT ([11]) which models an economy with the same cadre of agents (households, corporates, private banks, central bank and sovereign)

[18] This list aims to be indicative and not exhaustive. Other relationships that play a material role in the economy but are not listed are any type of rental / lease or insurance contract

	Households	**Corporates**	**Private Banks**	**Central Bank**	**Sovereign**	Category Sum
Real Assets	$\mathbf{As_h}$	$\mathbf{As_c}$			$\mathbf{As_s}$	$\mathbf{As}$
Cash	$\mathbf{Mo_h}$	$\mathbf{Mo_c}$	$\mathbf{Mo_b}$	$-\mathbf{Mo}$		0
Reserves			$\mathbf{Re}$	$-\mathbf{Re}$		0
Check Deposits	$\mathbf{De_h}$	$\mathbf{De_c}$	$-\mathbf{De_h} - \mathbf{De_c}$ (M1)			0
Term Deposits	$\mathbf{Te_h}$	$\mathbf{Te_c}$	$-\mathbf{Te_h} - \mathbf{Te_c}$ (M2)			0
Labor	$\mathbf{La}$	$-\mathbf{La}$				0
Tax Claims	$-\mathbf{Tx_h}$	$-\mathbf{Tx_c}$	$-\mathbf{Tx_b}$		$\mathbf{Tx}$	0
Debt	$-\mathbf{Lo}$		$\mathbf{Lo}$			0
Equity Inv.	$\mathbf{Eq_c} + \mathbf{Eq_b}$	$-\mathbf{Eq_c}$	$-\mathbf{Eq_b}$			0
Bond Inv.	$\mathbf{Bo_c} + \mathbf{Bo_b}$	$-\mathbf{Bo_c}$	$-\mathbf{Bo_b}$	$\mathbf{Bo_s}$	$-\mathbf{Bo_s}$	0
Net Worth	$\mathbf{NW_h}$	$\mathbf{NW_c}$	$\mathbf{NW_b}$	$\mathbf{NW_{cb}}$	$\mathbf{NW_s}$	$\mathbf{As}$

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