Open Risk White Paper

Deep-Linking Financial and Energy Accounting

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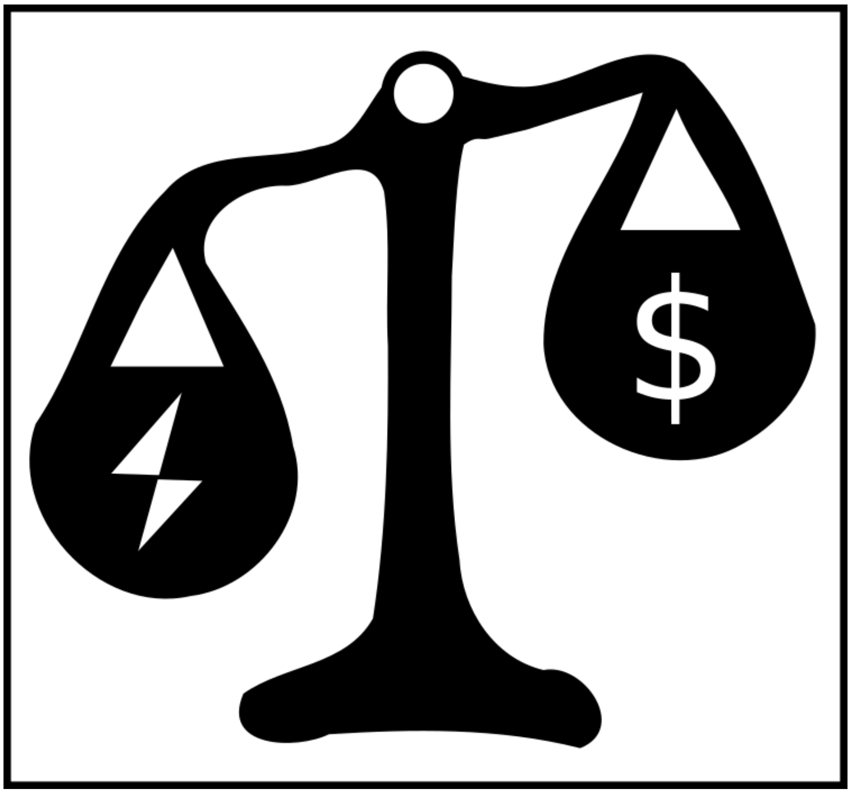

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Abstract

In this Open Risk White Paper we develop a conceptual framework for *integrated accounting* that produces (where possible) non-financial disclosures subject to the same double-entry balance constraints as those used to produce conventional financial statements and automatically ensures any additional conservation laws are satisfied. We identify the key ingredients required for such a rigorous integrated accounting framework, in terms of concepts, postulates and design choices. Our focus and concrete use case is built around *energy accounting*, keeping track on an entity's detailed energy footprint (primary inputs, transformations and waste generation) as an extension of its standard financial accounting and reporting. The central tool is the use of multi-dimensional double-entry bookkeeping which tracks quantitative information characterizing economic objects beyond their monetary values. This choice ensures the enforcement of both classic balance constraints and any applicable energy conservation laws. Further tools and techniques concern the aggregation and reporting of dual (monetary and physical) dimensions of an entity's accounting state. The framework is documented using mathematical notation.

Further Resources

- The Open Risk Manual is an open online knowledge base covering diverse domains of risk management.
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1 Introduction

1.1 Motivation

Les bons comptes font les bons amis

French Proverb

Accounting, along with a number of related sub-disciplines across finance, economics and risk management forms a pillar of the quantification of economic life. It is an essential *information technology construct* and it is difficult to imagine how complex modern society could function without *some* version of accounting. It has a very long history dating at least back to the clay accounting tokens of the Bronze Era. Its development is strongly intertwined with early information technology breakthroughs such as the adoption of script and seminal mathematical inventions such as numbering systems (including the use of zero and negative numbers). Later on, the art and science of keeping books got a major boost from the newly invented mass typesetting/printing technologies. In recent decades we have turned yet a new page of the accounting history book, with an ever evolving set of digital information tools such as databases and spreadsheets that are radically expanding the opportunities (and risks) associated with the proper keeping account of things.

In principle accounting helps mitigate the complexity of economic life. The core service aims to create more digestible, trustworthy and usable information artifacts. It is currently practiced globally in fairly diverse jurisdictions. It is the main channel for provision (disclosure) of financial information concerning the activities of large numbers of economic entities, both for-profit firms and the public sector. It is practiced widely both at the micro-level of individual entities (persons or companies) and at the macro-level of countries and governments. Within organizations, accounting starts with the business of internal (or management) accounting, which informs agents such as senior management, boards etc. Financial statements disclose externally select information about *economic resources* of the reporting entity, *claims against the entity*, and changes in both resources and claims. The scope of those disclosures is set in legal and regulatory reporting standards (e.g. IFRS). Depending on the type of entity, the purpose of such reporting is to provide useful information to existing and potential investors, lenders, other creditors and stakeholders of the organization but also the general public.

In modern times the economic complexity in terms of the number and type of exchanges that take place, has skyrocketed. Challenges to conventional accounting practice have been documented widely during the great financial crisis[1], including pathologies such as the reporting of immediate gains on securitisation which facilitated and motivated unsustainable levels of subprime lending, the ability to abuse internal estimates, which enabled firms to continue assuming risk and, finally, the abrupt eventual recognition of losses creating ripple effects across the economy. This work-in-progress nature of conventional accounting is also demonstrated by major recent revisions in specific domains such as IFRS 16 and IFRS 9.

The challenges that accounting must help address are not diminishing: The IPCC Working Group about Climate Change in its 2022: Impacts, Adaptation and Vulnerability Report (approved by no less of 195 member governments) states quite clearly: *The world faces unavoidable multiple climate hazards over the next two decades with global warming of 1.5C (2.7F). Even temporarily exceeding this warming level will result in additional severe impacts, some of which will be irreversible. Risks for society will increase, including to infrastructure and low-lying coastal settlements.* Dire as this warning might read, climate change is but one of the many so called Planetary Boundaries being crossed at rapid pace [2]. There is an increasing recognition that human economies as currently structured are not assured long term viability. The required systemic changes to accommodate life *within* such boundaries are commonly termed the **sustainability transition**. Suitable accounting systems will be an indispensable tool in this transition. The classic dictum *you can't manage what you don't measure* suggests that without treating sustainability metrics rigorously the transition will be difficult: susceptible to biases, undermined by blind spots and rife with *uninformed decisions*. Given the vast scope and intertwined nature and attribution of benefits and responsibilities in the global economy, another accounting dictum captures even more poignantly the nature of challenges ahead: *good accounts make good friends*. In a complex and contentious environment that carries heavy historical baggage, broad consensus that **sustainability accounting presents a fair and objective snapshot of reality** will be essential to maintain the global support needed for such a difficult transformation.

The collection and reporting of significant new facets of economic activity (collectively termed *non-financial data* or sustainability reporting) is already in full swing. An important emerging issue that is already much discussed is the ability of achieving management objectives on the basis of multiple and disconnected data points (e.g. financial versus environmental or social returns) [3]. New frictions (such as greenwashing) are already a concern. Making sure that new reported accounting dimensions are at least as reliably produced as existing ones and that such new facets are offering a consistent and usefully integrated view of economic activities should motivate more developments in this space.

With this motivation, sketching a deeper link between the old financial accounting schemata and the newly relevant sustainability dimensions is the objective for this white paper. We will be zeroing-in on a consistent framework for integrating financial and *energy accounting*. Focusing on integrated financial-energy accounts has three-fold motivation: The energy footprint of economic activity is i) very important in a sustainability context, ii) energy information can be readily quantified without losing much of its semantic content and iii) energy is not an economic externality (as are for example GHG emissions produced in the quest to tap energy sources) but a significant factor of production. More informed accounting of energy flows through the economy is thus a competitive advantage.

1.2 The fundamental role of energy

The economy can be thought of as a materials moving and processing system built on energy transformations. It is organized around distinct economic production/consumption entities (agents) that engage in a multitude of independent actions: extraction and conversion of resources, production (manufacturing), transport, trade and, finally, consumption. Thinking in terms of orders-of-magnitude, total primary energy extraction per year is of the order of 10^{14} kWh. This is roughly also the total GDP per year expressed in dollar units. Hence, figuratively speaking, a kilowatt-hour of energy is backing every dollar unit worth of value generated each year. Beyond such amusing back-of-the-envelope calculations it is true that a large fraction of industrial era *economic growth* was unexplained until the use of energy was introduced as an explanatory factor[4]. Such analyses provide more solid backing to the intuition that ever since the industrialization era the economy is on energy steroids. The unfortunate side-effect is that much of the human impact on the environment can also be, rather explicitly, traced to expanded energy use.

There is a raging debate about what amount of conventionally defined economic growth is compatible with sustainability[5]. Whatever the possibilities and plausible ranges of techno-economic scenarios, consistent tracking of material and energy flows as they percolate through the economy via vast supply-chain networks will require more holistic accounting approaches. In recent decades this task has started receiving significant attention across different disciplines: More comprehensive description of economic systems with a view towards sustainability implications include beyond pure monetary/financial metrics also material and energy flows, land use and of-course associated impacts (externalities or stressors). This is the subject of new disciplines such as industrial ecology and new paradigms such as industrial metabolism. In the domain of macro-economic models, there are new perspectives from so-called *stock-flow consistent models* that include material and energy stocks [6],[7],[8]. Extended accounting that includes non-financial (physical) units, in particular energy units is already practiced at the higher (macro) aggregation level by national statistical agencies in the context of *environmentally extended input-output models*. Physical energy flow accounts (PEFA) record the flows of energy from the environment to the economy (natural inputs), within the economy (products), and from the economy back to the environment (residuals). The usage of such frameworks has already provided insights and guides national policies.

1.3 Objective and Scope of the Paper

In more detail, our objective here is to explore *extensions of accounting systems at the micro level*, in particular the domain of corporate accounting and associated financial statements and disclosures. As mentioned sustainability reporting has already seen significant initiatives which aim to substantially extend the range of non-financial disclosures. But as indicated in[9], the assessment of current reporting practices shows that only a few companies provide clear linkages between sustainability and financial information. In part, this is explained by the lack of clear guidance and practical examples of the financial and non-financial information that should be connected. The importance of connectivity is also reinforced in Auditing Standards - ISA 720 *The auditors responsibilities relating to other information*, which requires the auditor to consider the *through line* from the financial statements to other information. The approach we take here is to look into the feasibility to **fundamentally link at a granular accounting level financial (monetary) phenomena with energy flows and transformations**. In this sense the scope of this work is quite a bit narrower than all-encompassing non-financial reporting initiatives that cover the entire range of sustainability considerations. To achieve this deep-linking objective we focus exclusively on measurable (quantitative) data and strive to mirror (to some extend) the recommended general structure of financial statements[10].

A separate objective is to present the framework in concrete and explicit mathematical notation. This allows for concise and unambiguous presentation, easy inspection of the elements of the conceptual framework and their linkages. In other related domains this might have been an obvious channel of communication but mathematics and accounting have a complex relationship. According to D.Ellerman (as quoted in [11]): *Double-entry book-keeping illustrates one of the most astonishing examples of intellectual insulation between disciplines, in this case, between accounting and mathematics.* Indeed adopting mathematical notation in communicating accounting principles (at least in parallel with existing textual statements) could help clarify what at time seems unnecessarily obscure expositions.

Our discussion will utilize stylized accounting entities. More fleshed out blueprints for specific sectors, in particular energy industries and financial intermediaries must fill-in the broad brush strokes with much intricate detail. The essential concepts are discussed at high level in this introduction and summarized in the Principles section 3.5 and Correspondence table 3.6. The more in-depth description that covers the same ground but adding mathematical and other detail is given separately in Section 2 .

1.4 Prior Work

Central to the generalization we explore here is the notion of diverse measurement qualities: quantifying the same economic phenomenon using different lenses (financial, energy etc). Each dimension being accounted for introduces a new *quality measure* of an underlying economic artifact. In conventional accounting all measured values populating an entity's accounts are **scalars** (that is, single values) expressed in financial (monetary / currency) units. Extending the accounting system introduces additional qualities or dimensions and the corresponding measurement methodologies required to establish reliably such values. Mathematically an *integrated accounting* space is a *multi-dimensional version* of the standard approach. Simplistically this means stacking alternate measurement views of the same reality, somewhat akin to a color photograph that is optically produced by overlaying three primary colors (red, green and blue). It is the different proportions of which give rise to the final visual perception. For example an extended account for an entity's inventory means that additional account slots would be populated with numerical values that not expressed in currency units (e.g. Euros) but other units (e.g., kilos, liters or Joules).

The possibility to generalize conventional scalar accounting into more dimensions and various potential uses has been recognized some time ago[12]. A more complete mathematical formulation of double-entry bookkeeping (DEB) in the context of an encompassing property accounting framework has also been proposed [13] but there is little apparent use of such formulations in practice. For our use case integrated accounting of energy flows must provide additional structure and constraints that will capture with the required fidelity the dual economic-physical phenomena of interest. Whether that can be achieved in practice with the universal adoption that financial based accounting achieved remains to be seen. In this white paper we zero-in on the structure and constraints that one must minimally address when thinking about accounting economic phenomena in relation with their energy footprint.

While physical phenomena and their measurements are entirely local in time and space, the economics of the same can be vastly *delocalized*. Thankfully, a viable set of conventions that bridges the two worlds has already been established in the context of energy input-output models (See [15] and references therein). The terminology used in energy input-output accounting models implicitly addresses some significant definitional difficulties, primarily by introducing the concept of *embodied energy*. Let us now briefly discuss these inter-related concepts that define energy with reference to its utility in the economy.

1.5 Energy Concepts

Before we can sketch what an integrated energy accounting book might look like, we need to discuss a small set of concepts from energy physics. The integration of energy considerations in the economic realm must reflect a few fundamental characteristics of this dimension when it is required to keep the representation of economic systems in alignment with their actual physical state.

- Energy is a quality and a quantity that is defined for any physical system and is one of the most fundamental notions in all of physics, irrespective of which spatial scale, material composition or nature of physical forces we might consider. For macroscopic systems composed of exceptionally large number of atoms energy is described using concepts from the sub-domain of physics named *thermodynamics*. With an eye towards economic applications, energy is defined simply as the ability to *accomplish useful work at human scale*.

- Reflecting the fact that energy is relevant in very diverse physical systems it appears in a variety of different guises, or species, types and forms: This includes chemical energy (e.g., what makes fossil fuels useful), electromagnetic energy (the secret of solar power), gravitational potential energy (the secret of hydropower), electric potential energy (what makes capacitors useful), kinetic energy (wind or wave power), nuclear energy and quite a few more esoteric forms. *Physical energy* such as the forms mentioned above can be measured with an apparatus that is adapted to the form of energy (e.g. a flow meter in the wall, a calorimeter etc.). The measurement is in physical units of energy. Despite the many species energy has a *single universal unit of measurement* (Joules).

- The first law of thermodynamics states that *for a closed system total energy across all its forms is conserved*. This means that the total energy of a closed system does not change in time but the *distribution* of energy in different forms may, and usually does, change.

- For an open system energy *conservation applies to the system plus its surrounding environment*. This means that any energy changes in the environment (from net inflow and outflow) must be the opposite of a system's own energy changes.

- The energy conservation law encodes a universal pattern (symmetry) that all physical systems respect: namely the fundamental laws of physics do not distinguish an arrow of time. Human-scale economic systems always display prominent *time asymmetry* as can be seen in the physical degradation and decay of artifacts and the dissipation of energy into less useful forms (heat). This is encoded in the second law of thermodynamics. For a closed system, the quality of energy, or the ability to do useful work *degrades over time* as the system on its own can only become more random / disorganized[1]

1.5.1 Primary and Secondary Energy

Primary energy is physical energy as first extracted from the natural world and brought into the economic system (as fossil fuel energy, solar energy, wind power etc). There is a major dichotomy between non-renewable and renewable forms of primary energy, but these variations are not important for the basic level of energy accounting we consider here. *Secondary energy* is physical energy that has been transformed to empower alternative economic uses (e.g., distribution via electricity networks). Secondary energy might be measured in physical energy units but has already *embodied* the energy lost in its transformation process from primary energy.

1.5.2 Useful and Final Energy

Useful energy is physical energy that is available to end-users to satisfy their energy needs. It is also referred to as *energy services demand* [16]. Transformation losses at the point of energy use means that the amount of useful energy is lower than the corresponding final energy requirement. E.g. mobility is based on the conversion of final energy into (useful) kinetic energy. The efficiency of that conversion varies. The *work* performed while producing goods and services is also termed useful energy. *Final energy* is the total physical energy (primary or secondary) that is required during the production *or* the consumption of *non-energy* goods and services. As we saw above, only part of final energy becomes useful energy while part of it is lost (degraded) as heat that dissipates into the environment. To the extend that final energy is used to produce non-energy goods and services, final energy is converted into the embodied energy of those goods and services.

1.5.3 Embodied energy

Finally we define this critically useful concept: *Embodied energy* of a product or service is the *historical aggregation* of all measured physical energy transformations that have enabled the production process. Herendeen and collaborators[17] first developed a theory of embodied energy in which the *energy cost* of a product is obtained by accounting for all the energy flows that have enabled its production process. It is essentially an information artifact that recognizes that processes that are spatially and temporaly dis-aggregated are actually causally connected from an economic perspective. The concept aims to reconstitute these linkages (e.g. towards measuring true efficiencies, attributing responsibility in fairer manner etc). Embodied energy is thus a purely social construct that addresses the dis-aggregated nature of economic processes (atomic activities that take place in extended supply chains and under the stewardship of distinct economic entities). There is no physical apparatus that can measure the embodied energy of any widget. Nevertheless every widget *might* have a label indicating its embodied energy. Just as a label indicating the caloric energy content of a food item measures the physical energy it contains (in biochemical form) a label indicating its embodied energy would be (a best effort) compilation of the energy it took to bring to the super-market shelf.

While physical energy cannot be destroyed (only transformed), embodied energy associated with artifacts can be *written off* when an artifact is de-recognized (removed from the accounts). The consumption (final demand) of goods or services with embodied energy releases the embodied energy of that service back into an abstract environmental account.

1.6 A minimal set of energy qualities

The energy qualities that will be measured and reported play a central role in the formulation of integrated energy accounting. Already the above short discussions allude to a number of options addressing both the physical dimension (forms of physical energy) and its embodied dimension (usage within economic systems). The minimal design must on the one hand capture real energy flows, thereby linking with actual environmental states and impacts and on the other hand incorporate energy provenance as this is essential for fair attribution of benefits and responsibilities. We will use the term physical energy to denote the first measurable while embodied energy is the second measurable.

[1] We will not expand on the possibilities of incorporating energy efficiency qualities in accounts, but since the early work of [14] there is an important branch of lifecycle analysis in energy that incorporates *exergy*

The conversion from physical to embodied energy that happens in production and consumption can be thought of as an fictitious chemical reaction which links the social dimension with physical reality. Significant refinement of both energy dimensions is in-principle possible. Physical energy can be tracked along its more specific forms, which provides links to specific technologies. Embodied energy can be decomposed into useful energy and various forms of waste which reveals efficiencies at a deeper level. Yet these refinements follow the same conceptual accounting pattern and introducing them here would overburden the discussion.

2 Integrated Internal Accounting

Having sketched at high level the conceptual framework in the introduction section 1, we turn now to fix concepts more concretely and using mathematical notation. The journey will culminate in a blueprint of integrated accounting reports in section 3.4 that shows that the generalization of of contemporary accounting standards is quite feasible.

2.1 Accounting Entity Definition

At the legal level an accounting entity asserts *ownership* (rights) over a list of valuable artifacts (economic resources) via various enforceable *claims*. It also engages in binding *contracts* that stipulate terms for future economic exchanges with other entities. An entity's operations have measurable implications (either directly as a producer/consumer or indirectly as a facilitator/intermediary) for energy flows in the economy. Its internal accounting and external reporting aims to faithfully record and report these flows. Towards that end we assume an internal information management system that captures the entity's financial and energy accounting activities. The system runs continuously and updates its state with incoming information. The backbone of this system is an inventory $\{i_t\}$ of economic items and relations that points to quantitative and qualitative information about the entity's owned economic objects or artifacts. For concreteness we may think of this inventory as a database that describes anything from enumerations of tangible physical assets, procurement and employment contracts to leases, other financial contracts, intellectual property, tax obligations etc. The precise scope of this economic inventory can only be defined once the organizational nature and objectives along with the intended use and practical constraints of integrated accounting have been defined in more detail. The above mental exercise primarily aims to prepare us for the important next step the introduction a suitable *accounting state space*.

2.2 Integrated Accounting State Space

At any given time t the *economic relationships inventory* or portfolio (denoted as $\{i_t\}$) might be thought as a large but finite list (or enumeration using unique identification numbers) of various economic objects (physical artifacts, financial contracts etc). This inventory defines the perimeter or scope of the entity in legal, physical (in particular also energetic) and financial terms. It is not just the inventory of tangible, physical artifacts with directly measurable energy profile: For many entities (e.g. financial intermediaries) this inventory might be an almost entirely virtual construct.

Starting with the above sketched economic inventory $\{i_t\}$, the accounting system summarizes this sprawling, unstructured and inhomogeneous information. Various present and past economic facets of an entity, along with any hard-wired constraints about future activity in terms of contracts is squeezed into a well defined (and ultimately drastically simplifying schema of *numerical values*: the set of accounts $\{a_t\}$. In accounting jargon the set of accounts might be identified as the *general ledger*, which is a collection of labels and data containers that form the scaffolding for the actual accounting system.

The relationship of the accounting system with the underlying information pool is a *surjective function*. Namely it maps the economic inventory space $\{i_t\}$ to the set of numerical containers (accounts) $\{a_t\}$, or symbolically a function $a = f(i)$. In other words, every economic significant relation of the entity must be accounted for *somewhere* in its list of accounts, though not necessarily on a one-to-one basis. In practice accounts reference collections of similar economic objects whose value can be established using the same methodology. Needless to say, the restrictive, purely quantitative format of these accounts loses enormous amounts of information. In external reporting this is compensated in part with additional disclosures (notes). The simplicity introduced by the homogeneous quantification allows for relatively easy analysis and comparison of various entities. The creation of accounts from the underlying legal/economic reality is termed *recognition*. The *meaning* of accounting values stored in the list of accounts is flexible and open ended. The major focus of all modern accounting is to construct maps from economic states and state changes to *financial values* as we will see in section 2.3. This is achieved by specifying a *measurement procedure*, that is, a process for assigning numerical values from underlying economic activity data.

In a well developed *energy accounting* framework the objective is to provide also a quantified map of the *energy profile* into numerical energy measures. An energy accounting system will have a parallel recognition process to assess the energy profile of economic activities which can be based, e.g., in the sectoral methods of IAE or further refinements [2]. An *integrated general ledger* will in general have additional accounts versus a purely financially oriented one such as an environmental account needed to *close* and balance the accounting system.

In integrated *energy* accounting the quantification process of an entity's economic relations inventory creates accounts that hold *value tuples* (or *vectors*) where each account tuple[3] holds both the standard financial *and and* additional energy related measures. The *dual* energy and financial approach can be seen as an expanded accounting universe that captures two important and related but distinct dimensions of the state space of economic entities. The setup can be generalized to use as many qualities of measurement as necessary. The use of monetary (financial) value and two energy qualities (physical energy and embodied energy) is a minimal extension but is certainly not an exhaustive characterization. In the sequel we will use a general number q (number of qualities measured) to indicate this generalization potential, but all concrete expressions in this paper imply the $q = 3$ case where $q = 1$ is the monetary dimension, $q = 2$ is the physical energy dimension and $q = 3$ is the embodied energy dimension.

As a result of the accounting recognition process the *accounting state* of an entity is a finite list of N numerical tuples V_t^a representing *accounting values* (measurements) of each account at time t. These values will form the quantitative backbone of the integrated accounting system. The accounting space of a system with N accounts is thus a type of vector space $R^N \times R^N \ldots \times R^N$ produced by the q-fold Cartesian product of R^N where q is the number of *qualities* being accounted for. [4] Let us turn now to discuss how these placeholder accounts are filled with tuples of values V.

2.3 Measuring Account Value Tuples

The financial side of measurement is simply the set of existing valuation methodologies V_M used in current accounting systems. This comprises maps or functions from items in the economic inventory to monetary units (*the value of economic objects*). Recall that these valuations are applied to homogeneous collections of similar property rights, financial contracts etc. Correspondingly, energy measurements of different energy qualities are the maps $V_E, V_{\mathcal{E}}$ from aggregated homogeneous economic objects to energy units (*the energy profile of things*). Symbolically the overall measurement process reads:

$$V_M(a_t) = \text{Financial value of accounts at time t} \tag{1}$$
$$V_{\mathcal{E}}(a_t) = \text{Physical energy of accounts at time t} \tag{2}$$
$$V_E(a_t) = \text{Embodied energy of accounts at time t} \tag{3}$$

Measurement procedures, methodologies and functions are in practice quite complicated and there are typically a number of *incompatible* alternatives capturing different aspects. Financial valuation is a smorgasbord of conceptually quite diverse philosophies (just to mention: the chasm between historical cost and fair value approaches that necessitates complex ring-fencing of reports so as to ensure apples-with-apples comparisons). Measurements may thus involve a significant number of granular data points (e.g., schedules of cash flows) and subjective evaluations (e.g. scenario based risk assessments) (Principle 3) 3.5. Importantly, though the basic *structure* of the integrated accounting framework does not depend sensitively on the *details* of measurement.

As discussed in the introduction, the minimal requirement is to account for two energy species:

- Physical energy which is the direct input of energy from the environment (either as primary energy or as secondary (transformed) energy and which can be measured (in principle) directly using physical probes but in practice may rely also on a hierarchy of indirect methods (tables and formulas deriving energy from other measurables). The market value of physical energy is one of the readily available means to measure physical energy (using a conversion rate using prices). Tangible examples of such measurements are the IAE methodologies and datasets as mentioned above.

- Embodied energy which tracks the cumulative energy utilization in the process of producing, trading and consuming goods or services. This is a virtual construct that accumulates amounts of physical energy that have been used for the production of a good or service. Thus, embodied energy measurement, while a new specification, essentially piggybacks on physical energy measurement methodologies for primitive numerical inputs. Measuring embodied energy in this manner is already practiced in Life Cycle Analysis applications.

[2]IEA collects energy efficiency data through an annual questionnaire that compiles energy consumption and activity data for various end uses, sub-sectors and modes/vehicle types across four sectors: residential, services, industry and transport.

[3]A tuple is simply an ordered list of values, e.g. ($\$102, 30\,kWh, 10\,kWh$)

[4]While the appearance of this vector space might suggest a certain uniformity of the dimensions, given the different units of each operations *between* dimensions are in general not allowed.

In practice one might segment the inventory of accounts into those holding energetically significant values and structure reports to avoid sparse tables. Given this conceptual construction accounts have placeholders for all measured values. In fact the discipline of justifying why any given account has or doesn't have a given energy profile and how that balances with all other accounts is an important benefit of integrated accounting.

The energetically relevant physical inventory need not be owned by the entity. E.g., a *right to use asset* from a lease contract may well be an important item from an energy point of view. The relevant inventory need not even be physically tangible. E.g. intellectual property or other intangible assets generated through intensive energy processes might also be assigned substantial embodied energy. Schematically the measurement of all accounts of an entity produces a matrix of values as follows:

$$V = \begin{bmatrix} V_1 & V_2 & \dots & V_N \\ \hline v_{11} & v_{12} & \dots & v_{1N} \\ \dots & \dots & \dots & \dots \\ v_{q1} & v_{q2} & \dots & v_{qN} \end{bmatrix} \tag{4}$$

where the first row is just the label of the account. Each account has a *column* of measured qualities v_q^a. For concreteness we might think of a stylized entity with the following list of accounts:

V1:	C	Cash
V2:	F	Plants: Solar Panel Array and Widget Factory
V3:	S	Energy Stock
V4:	M	Raw Materials
V5:	I	Widget Inventory
V6:	**P**	Accounts Payable
V7:	**L**	Bank Loan
V8:	**K**	Equity

where in principle all accounts have three associated measurements (expressed in their respective units):

$$V = \begin{bmatrix} C & F & S & M & I & \mathbf{P} & \mathbf{L} & \mathbf{K} \\ 120 & 1000 & 300 & 150 & 600 & 0 & 0 & 2170 \\ 0 & 0 & 500 & 0 & 0 & 0 & 0 & 500 \\ 0 & 100 & 0 & 600 & 700 & 0 & 0 & 1400 \end{bmatrix} \tag{5}$$

2.4 Integrated Double-Entry Accounting

Double-entry accounting or double-entry bookkeeping (DEB) aims to automatically *balance* a set of accounts:

$$\sum_a^N V_t^a = 0 \tag{6}$$

which in words means that the summation of all account values is zero at all times. Mathematically the DEB design has been described elegantly using group theory (the so-called *Pacioli Group*[18]) or a special type of matrix space (Balance Vectors whose sum of values is always zero)[11]. The balance requirement generalizes naturally to multi-dimensional sets of measured qualities. A multi-dimensional DEB accounting framework has already been discussed [13] in the context of property theory. In the general case the measurement matrix V capturing the values of all accounts satisfies the *balance equation for all measured qualities q*:

$$\sum_a^N V_{q,t}^a = 0, \text{ for all } q \tag{7}$$

Thus while in standard accounting balance equations only involve $q = 1$ (the financial leg), more general systems require as many balance equations as the range of q. If we write out explicitly the legs of the balance equation 7 for our system where we track financial value, physical energy and embodied energy we have:

$$\sum_a^N M_t^a = 0, \qquad \text{Total Financial Value Balance} \tag{8}$$

$$\sum_a^N \mathcal{E}_t^a = 0, \qquad \text{Total Physical Energy Balance} \tag{9}$$

$$\sum_a^N E_t^a = 0, \qquad \text{Total Embodied Energy Balance} \tag{10}$$

In analogy with the balance vectors of standard financial accounting, matrices with their *row sums equal to zero* are called *balance matrices*. They are the mathematical foundation for an integrated accounting system.[5] This special type of mathematical space forms a *hyperplane* in the unconstrained space of signed real values $R^N \times R^N \dots R^N$. Further constraints are placed by any additional physical conservation laws as we will discuss below.

2.5 Assets and Liabilities

The decomposition of accounts into assets and liabilities determines the interface of the entity with the outside economic world as an internal set of (by convention) positively valued economic resources (assets) and a matching set of negatively valued external economic objects (liabilities). Equation 7 written down for an explicit set of assets and liabilities leads to the standard *balance sheet equation* which equates assets, liabilities (including equity) at all times. This is implemented as a set of conventional signs (plus or minus) that prescribes how the values of different accounts is to be added across the balance equation. We indicate this split by segmenting the index a of accounts $V_{q,t}^a$ in two sets of assets and liabilities (plus equity) and using boldface to denote liabilities:

$$V = \{V^A, \mathbf{V}^L, \mathbf{K}\}, \qquad \text{Separation into Assets and Liabilities} \tag{11}$$

$$\sum_A V_{q,t}^A + \sum_L \mathbf{V}_{q,t}^L + \mathbf{K}_{q,t} = 0, \qquad \text{Balance of Assets and Liability Value Rows for all } q \tag{12}$$

where A, L are index sets ranging over the assets and liabilities respectively. If we write out explicit assets and liabilities along the three measurement dimensions we have:

$$\sum_A M_t^A + \sum_L M_t^L + M_t^K = 0, \qquad \text{Asset-Liability Financial Balance} \tag{13}$$

$$\sum_A \mathcal{E}_t^A + \sum_L \mathcal{E}_t^L + \mathcal{E}_t^K = 0, \qquad \text{Asset-Liability Physical Energy Balance} \tag{14}$$

$$\sum_A E_t^A + \sum_L E_t^L + E_t^K = 0, \qquad \text{Asset-Liability Embodied Energy Balance} \tag{15}$$

The adoption of the financially implied asset-liability decomposition to express energy accounts is the expression of Principle 2 (alignment of scope) 3.5.

2.6 Accounting Transactions

Over time the entity's inventory of economic resources and relations will change. E.g., an entity's claims of ownership evolves due to operating, financing and investment activities materialized either immediately with exchanges or via longer term contracts. New items N_t will be inserted in the inventory i_t, existing ones will be modified and some old items O_t will expire or be canceled (exchanged, depreciated, destroyed etc) and removed. In turn, the structure and values of the accounting system $\{a_t\}$ must be *updated* to reflect the new reality. Such evolution can be handled by decomposing the inventory into subsets using the union and set-difference operations: $I_{t'} = I_t \cup N_{t'} \setminus O_{t'}$. Hence as a result of changes in the economic and energy state of an entity there is a corresponding accounting state migration from a_t to $a_{t'}$ and corresponding changes in measured values. The mechanism for recording accounting state mutations are *accounting transactions*.

The classic transaction concept can be readily generalized to handle dual or higher dimensional measurements. In order to satisfy the design of double-entry bookkeeping, accounting transactions must themselves satisfy balance conditions for

[5] NB: These matrices are quite distinct from accounting matrices representing transactions *between* accounts

all measured qualities. Mathematically both accounts *and* transactions are *balance matrices* with their row-wide elements always summing up to zero. Each transaction is a multi-dimensional array of values T_q^a, where a ranges over N accounts and q ranges over the measured dimensions (qualities). The values of this array (*the transaction measurements*) are derived following similar methodologies as per the measurement of underlying accounts.

$$T = \left[\begin{array}{cccc} \Delta V_1 & \Delta V_2 & \dots & \Delta V_N \\ \hline t_{11} & t_{12} & \dots & t_{1N} \\ \dots & \dots & \dots & \dots \\ t_{q1} & t_{q2} & \dots & t_{qN} \end{array}\right] \tag{16}$$

The balance requirement for transactions can be expressed exactly as for accounts:

$$\sum_a^N T_q^a = 0, \text{ for all } q \tag{17}$$

where a ranges over the N accounts and q ranges over all qualities. Note that these balance constraints apply *separately* for each horizontal (row) dimension. Given these update values, the next accounting state of the entire system can be obtained simply by a *numerical addition* of transaction matrices T to existing values V:

$$V_{q,t+\epsilon}^a = V_{q,t}^a + T_{q,t}^a, \text{ Posting a transaction at time t} \tag{18}$$

In our stylized booking system the result of posting a transaction $T_{q,t}^a$ at time t is to update accounts at that exact *instant* (we assume the update only takes up infinitesimal time ϵ). As an example, the following transaction matrix

$$T = \begin{bmatrix} \Delta C & \Delta S & \Delta \mathbf{K} \\ -x & x & 0 = x - x \\ 0 & \delta\mathcal{E} & \delta\mathcal{E} \\ 0 & 0 & 0 \end{bmatrix} \tag{19}$$

expresses spending a cash amount x to acquire x worth of physical energy stock $\delta\mathcal{E}$ (which being at market value has zero impact on PnL). The transaction also illustrates attribution of that physical energy to equity.

Any stock account matrix that satisfies an initial balance equation and mutates only using balanced transaction preserves that balance into a final equation.

$$\sum_a^N V_{t-1}^a = 0, \text{ Balance at time t-1} \tag{20}$$

$$\sum_a^N V_t^a = \sum_a^N \sum_\tau T_\tau^a + \sum_a^N V_{t-1}^a \tag{21}$$

$$= \sum_\tau \sum_a^N T_\tau^a + 0 \tag{22}$$

$$= 0, \text{ Balance at time t} \tag{23}$$

Taken together, the balance conditions 7, 17 and the update equation 18 circumscribe the essential mechanics of the accounting system and they express Principle 1 3.5. The integrated (financial and energy) state of an entity is updated by applying atomic transactions that modify all affected measured or attributed qualities of an account preserving balances across all measured dimensions.

2.7 Transaction Types

There is a vast array of possible transactions already before we extend the system to capture energy profiles. We might distinguish internal versus external transactions (that involve other economic parties). External transactions will involve at least one liability account. Internal transactions might capture real activity (production or consumption) by adjusting internal accounts of stocks and inventories or be purely bookkeeping operations. External transactions with other economic entities might be cash or credit based. Abstract transactions with environmental accounts can represent non-monetary extraction of resources. Many transactions are purely contractual in nature (financing, investments or distributions to equity). All of the above distinct types of transactions affect the different measured values (financial, physical energy, embodied energy) as encoded in specific transaction values $T^a_{q,t}$. The list of non-zero values and the implied underlying measurement methodology ultimately determines the type of transactions. Some important transaction categories that highlight the different ways qualities of energy might percolate through an entity's accounting system:

- *Energy Trading Activity*: In principle all purchasing or selling of goods and services from other economic entities involves exchange of some physical and/or embodied energy. E.g., any material goods transaction changes the stock of embodied energy (which must be reflected on both asset and liability sides). Purchasing or selling physical energy changes accounts in a similar way. Accounting for physical energy might need introducing dedicated accounts that reflect the storage of physical energy in fuel containers or other forms of energy storage.
- *Goods and Services Production Activity:* These transactions express the transformation of physical energy into embodied energy in the process of production. As discussed, the latter includes useful energy *and* losses. Any production activity, whether captured explicitly as an inventory increasing transaction or implicitly via income / expenses transactions will hit some physical energy accounts with a decrease and the embodied energy account with an increase. Such transactions are special as they must satisfy physical conservations laws in addition to the accounting balance equations.
- *Contractual Activity*: Pure contractual activities such as financing or investing do not involve directly measured amounts of physical of embodied energy (unless of-course they concern energy contracts) but will change the attribution of physical and embodied energy to accounts.
- *Energy Extraction or Release from/to the Environment*: Primary energy producers extract from the environmental account and consumers release into the environmental account.

The above categorizations are just indicative. We assume that there is a complete taxonomy (Principle 1) 3.5: all activity is captured with such transactions.

2.8 Measurement versus Attribution

The balance equations we have seen above have the general form of *conservation laws*, governing flows in and out from a defined system. For example *Kirchhoff's first law* governing the conservation of charge flows in and out of a network node is exactly of this form. There is though subtlety involved in the use of these accounting balance equations to express energy conservation laws. Before we do that we need to discuss the important role of *attribution* to accounts.

An important aspect of financial accounting is that transactions measure and modify *directly* the financial value of the *specific* affected assets and liabilities. In general without any reference to the value of the remaining assets or liabilities. An important exception is accounting equity, which is *not* measured directly but rather *attributed* the residual value (value of assets minus value of other liabilities). While this attribution only happens once (at inception) and for one account (equity) and is thereon automatically preserved by DEB principles, it is an operation of fundamental importance: In contrast, e.g. with electric charge conservation that flows into and out of an electric circuit node, this attribution operation implies that there is no monetary charge conservation in the basic balance equations. If one insists to measure equity using market values one will discover that the balance does not hold [6].

The attribution pattern plays an expanded role in energy accounting because several more typical accounts cannot be associated with directly measurable energy qualities. Most representative of this phenomenon are cash accounts and general purpose loans. This is maybe clearer if we walk through some transactions: An entity obtains a loan from an intermediary, the cash account is incremented alongside a matching loan liability. There is no physical or embodied energy exchange taking place at that time, nor any strings attached: the obtained cash can be exchanged against the entire menu of economic artifacts available in the economy. At a later date, the entity may use *some* cash to obtain, e.g., equipment, raw materials, energy stock, pursue investments in other entities etc. In this process financial value shifts from cash to other owned asset accounts. Such transactions do bring inside the entity measurable physical or embodied energy that is added to the relevant assets accounts. Yet there can be no *direct* attribution of these energy adjustments to the above loan. The cash used may have been (in part) from other liabilities or an equity capital injection. The causal link between cash and loan is tenuous: Generic financing activities (including equity funding) provide discretion in the use of funds hence the linkage and balancing of energy measurements across asset and liability sides cannot (in general) be direct.[7]

[6] Except in extremely idealized perfect and complete market universes where everything is a tradeable and there is no arbitrage

[7] The exception might be special liabilities that preserve causal links and trace the use of funds: E.g. Accounts Payable or specific Project Finance accounts.

General liabilities are broad based enablers of the entity (they underwrite a privilege to use energy towards economic goals) which means that when balancing energy measurements on the asset side they must be attributed *some* responsibility. How would one go about this attribution? This is clearly an additional methodological choice one must make. The precise attribution methodology is not fixed a-priory: Liabilities of the organization will be attributed energy values in accordance to some attribution schedule. The overarching requirement is that the energy accounts balance. What options are available? As a starting observation equity *must* be attributed any residual energy just like it is attributed residual wealth: Namely if an entity has no other liabilities (which is conceivable), the entire energy footprint of an entity must be attributable to equity (Principle 6) 3.5.

In the presence of other liabilities there is a question of how to balance (attribute) total asset energy measurements on the liability side. Different approaches are conceivable:

- Attribute total energy to equity irrespective of other liabilities. This is simple enough but may create very distorted incentives.
- Residual attribution to equity after accounting for liabilities where the use of financing proceeds is known and tracked (ring-fenced liabilities).
- Attributing energy to generic liabilities and equity pro-rata their financial value.

Clearly there are many many variations and fine tuning possible e.g. one might argue that the longer the maturity of liabilities the more responsible. Fair attribution is a major outstanding issue (Principle 4) 3.5 Mathematically we may write an attribution methodology for each liability a as:

$$\mathcal{E}_t^a = f_t^a \mathcal{E}_t^T = \sum_A \mathcal{E}_t^A, \, a \in L \tag{24}$$

$$E_t^a = f_t^a E_t^T = \sum_A E_t^A, \, a \in L \tag{25}$$

$$\tag{26}$$

where $\mathcal{E}_t^T, E_t^T$ are the total physical / embodied energies of the asset side and f_t^a is a set of allocation factors that satisfies

$$\sum_a f_t^a = 1, \, a \in L \tag{27}$$

An important practical implication of using such a global attribution schedule that takes into account the presence of multiple generic liabilities is that in order to maintain the standard state space behavior of double-entry accounting systems these attribution factors will have to be (in-principle) *recomputed* in connection with each accounting transaction.

2.9 Expressing Energy Conservation Laws

Given the stylized description of an entity's financial and energy state that we have developed so far how can we express the energy conservation law? Articulating an energy conservation law (and more generally physical constraints such as stoichiometric balance equations) relies on a suitable definition of the system's perimeter. An accounting entity is not a closed system in either the physical or the embodied energy sense. The external environment is represented by the set of all other economic entities *and* environmental accounts. Identifying this external environment helps close the system (Principle 5) 3.5

Energy conservation in our context is a statement about *stylized energy flows* across the entity's economically defined perimeter as expressed through the asset and liability decomposition. It is captured by the constraints on transaction values affecting physical and embodied energy:

$$\sum_a^N \delta \mathcal{E}_t^a = 0, \qquad \text{Physical Energy Flow Balance} \tag{28}$$

$$\sum_a^N \delta E_t^a = 0, \qquad \text{Embodied Energy Flow Balance} \tag{29}$$

$$\sum_{a \in A} \delta \mathcal{E}_t^a + \sum_{a \in A} \delta E_t^a = 0, , \qquad \text{Physical to Embodied Energy Transformations} \tag{30}$$

The first equation is simply the DEB constraint for physical energy transactions. It states that the total physical energy change on all accounts (assets and liabilities) is zero. This covers among other any physical transfer of energy between asset accounts that is assumed to happen without losses. The second equation mirrors the above discussion for embodied energy. The constraint enforces in particular that exchanges of material objects preserve the embodied energy count. In the absence of energy transformations theses two pools are conserved separately and there is no need for any other constraint.

The third equation recognizes that energy transformations between energy qualities are possible and sets a constraint that the transactions representing such transformations must preserve total energy on the asset side of the entity. This automatically preserves total energy also on the liability side.

2.10 Expressing the Flow of Entropy

Notice how (as expected) energy conservation does not prescribe the direction of energy flows. The constraint allows embodied energy to be converted to physical energy (provided the balance is kept) which is makes no sense. Not only is embodied energy simply a label, it encompasses energy losses to the environment that are in general non-reversible. If we need to explicitly express this one-directional flow of entropy we need to isolate accounts and transactions that express *pure energy conversions* from one energy form to another (as opposed to horizontal transfers between accounts). For example consider an inventory of materials which, along with some energy stock, is converted into widgets. The energy stock is depleted as work and heat loss, while the widget embodied energy is augmented. In such pure conversion transactions $\{C\}$ the entropy increase is expressed simply as

$$\sum_{a \in C} \delta E_t^a \geq 0,, \qquad \text{Embodied energy can only increase} \qquad (31)$$

3 Blueprint for Integrated Accounting Statements

Let us recap the journey thus far before we discuss how one might construct statements or external reports on the basis of an the integrated accounting framework. We started with an economic relations inventory $\{i_t\}$ of economic objects hat circumscribed the physical and financial perimeter of an entity. We measured a set of q-dimensional accounts $\{a_t\}$ that assigned matrix-valued qualities (financial value, physical and embodied energy value). We discussed accounting transactions and how those preserve balances along all measurement dimensions, including the enforcement of energy conservation.

The construction up to this point was a granular (low level) accounting system. It is the hypothetical raw material for generating more digestible information for internal and external use. Transactions are happening continuously and at random times. Each one contains a matrix worth of values that update the state of a potentially very large number of accounts. This system contains volumes of information that are not easily inspected or understood. How can we start from this real-time, high-frequency and granular internal booking / management information system to produce summarized reports at *regular intervals* for human consumption? Our end goal is a set of high-level reports that more or less mirror currently practiced reporting standards. Creating conventional looking integrated statements involves many additional choices beyond those we already adopted for delineating the underlying accounting system. Conceptually the *report generation sequence* and its generalization to multi-valued reports works roughly as follows:

1. Integrate (accumulate) a *journal* of transactions which forms the basis for period-on-period *change equations*. The generalization to multi-valued journal is straightforward.

2. Define intuitive *aggregate variables* (summations) for both stock type variables (assets and liability taxonomies) and their deltas or flows (e.g. sales, expenses etc). Energy considerations will require add a number of new aggregations (e.g. purchased or extracted energy etc)

3. Present aggregate stock accounts (the statement of financial position) and highlight accounting identities. Energy values can be presented next to monetary values. Applicable conservation laws can be expressed as additional identities.

4. Identify subsets of stock accounts that will see specific *explanation of mutations* (e.g the mutations of the cash account or the equity components).

5. Explain these specific stock mutations using the set of corresponding *aggregated flow accounts*. Energy flow statements can be developed in analogy to the cash flow statement.

6. Optionally, split flow statements according to underlying measurement methodologies.

Let us see how to express the above recipe in mathematical notation.

3.1 Connecting Journals and Ledgers

The key tool is the *journal of transactions*. A journal is accounting jargon for a time-ordered list of transactions $J = \{T\}$ that occur during a defined period of time $[t_{start}, t_{end}]$. Actual journals include contextual information beyond numerical values but we focus here on the purely numerical elements. We use the term throughout to mean *compound journal entries*, that is, entries that affect more than two accounts at the same time.

The temporal intervals for reporting are usually built over equal-time periods, e.g., a temporal grid $\{t_1, t_2 = t_1 + \delta t, \ldots, t_n = t_1 + n\delta t\}$. The journal of transactions between times $t-1$ and t is the set:

$$J_{t-1,t} = \{T_\tau, \tau \in [t-1, t]\} \tag{32}$$

A journal is applied to accounts by processing (posting) all its entries in temporal sequence (from oldest to newest) as per equation 18. Processing a transaction (journal entry) transforms one set of valid bookkeeping statements (the ledger) into another set of valid bookkeeping statements (the updated ledger). Other terms used for this update procedure are roll forward or movements of accounts. Applying the journal produces the *accumulated set* of account mutations over an interval $[t-1, t]$. Hence beginning account balances plus additions minus subtractions become ending account balances.

$$V_t^a = \sum_\tau T_\tau^a + V_{t-1}^a = J_{t-1,t}^a + V_{t-1}^a \tag{33}$$

Equation 33 can be seen as describing a discrete dynamic system that is driven by the inflow of transactions which modify the system's state.[8] For each affected account we have explicitly:

$$\begin{pmatrix} \text{Financial Value at t} \\ \text{Physical Energy at t} \\ \text{Embodied Energy at t} \end{pmatrix} = \sum_\tau \begin{pmatrix} \text{Financial Value Change at } \tau \\ \text{Physical Energy Change at } \tau \\ \text{Embodied Energy Change at } \tau \end{pmatrix} + \begin{pmatrix} \text{Financial Value at } t-1 \\ \text{Physical Energy at } t-1 \\ \text{Embodied Energy at } t-1 \end{pmatrix} \tag{34}$$

where τ ranges over all the individual transaction times.

A journal of transactions is preserving balances at fixed reporting timepoints rather trivially as this follows from the mathematical properties we have imposed on accounts and transaction matrices: An accounts matrix V that satisfies an initial balance equation 7 and mutates only using a journal of balanced transactions (equation 17)

$$\sum_a^N V_{q,t}^a = \sum_a^N J_{t-1,t}^a + \sum_a^N V_{q,t-1}^a, \tag{35}$$

$$= \sum_a^N \sum_\tau T_{q,\tau}^a + \sum_a^N V_{q,t-1}^a \tag{36}$$

preserves its balance as expressed by equation:

$$\sum_a^N V_{q,t}^a = 0\,. \tag{37}$$

3.2 Aggregation of Stock Accounts

External reporting typically operates at a fairly high level. Accounting states and transactions reported are grouped (*aggregated*) according to predefined taxonomies. In the aggregation process elements across the accounting state are summed up. The basic equation for an aggregation (or roll up) of a subset of accounts b into an *aggregate account* is simply the addition of values over that subset:

$$S_t^b = \sum_{a \in b} V_t^a \tag{38}$$

In words, a coarse-grained set of accounts b_t is constructed by grouping elements from a fine-grained set a_t. This coarse-grained grouping is also surjective: each granular account must be included (once) somewhere in the higher-level

[8] Obviously we are glossing over many practical details, for example the requirement that accounts have sufficient balances.

representation. Aggregation applies to both asset and liability accounts. For example, a cash account might accumulate all cash-like balances (whether bank notes, bank account balance, other cash-equivalents etc). A general inventory account might accumulate the value of all internal product inventories, no matter what their nature etc. Current / Non-Current liabilities provide a rudimentary term structure for aggregated amounts owed. This aggregation is also applicable across measurable qualities: The physical and embodied energy of a set of granular accounts may also be summed up into a reported total.

3.3 Aggregation of Transactions into Flows

Similar to the aggregation of accounts we may summarize similar transactions into aggregated *transaction flows*. Mutations (deltas ΔS) of aggregated stock accounts are induced by the aggregated transaction flows F^b:

$$S^b_t = S^b_{t-1} + \Delta S^b_{t-1,t} \tag{39}$$

$$\Delta S^b_{t-1,t} = \sum_{a \in b} \sum_{\tau} T^a_\tau = F^b_{t-1,t} \tag{40}$$

The flow accounts F^b group similar types of transactions: E.g., a sales flow account might accumulate all realized sales transactions in the journal. An energy inflow account might accumulate all transactions where physical energy flows into the entity through purchases or primary extraction. Both aggregated stock and flow accounts satisfy the balance equations. The key difference between them is that stock accounts have an initial balance at the start of a period (that is typically non-zero). In contrast, temporary flow accounts e.g. profit and loss over a period, are simply the sum of mutations from transactions without any natural non-zero initial balance. Sometimes in accounting literature stock and flow accounts are bundled together under the name of accounts (they are all numerical matrices that *balance* after all) but keeping in mind their different role in equation 40 helps with understanding the construction of specific *financial statements*.

3.4 Selecting what to explain

The neat stock-flow setup of the previous step *is not* what is being reported in standard practice. The final step of the process which leads to actual *financial reporting* as currently practiced is as a function (map) R that selects from the aggregated accounting states and flows S, F a smaller number of accounting elements. This creates a narrower, focused, set of stock elements $\hat{S}$ (with the complement unfocused group denoted $\tilde{S}$) and corresponding focused flows $\hat{F}$ and shadow flows $\tilde{F}$.

One indispensable focus account that receives royal treatment in standard reporting is the equity account. Another important focus point is the cash account. Focus accounts are first among equals and receive their own change statements. Such statement are constructed simply by reporting the composition of the corresponding flows. In principle any of the aggregated stock accounts could be explained this way at the expense of a larger number of statements.

As a result of the above choices the basic *financial reporting equations* looks in general as follows:

$$\sum_b \hat{S}^b_t + \sum_b \tilde{S}^b_t = 0, \quad \text{Aggregated Balance Equations} \tag{41}$$

$$\hat{S}^b_t = \hat{S}^b_{t-1} + \hat{F}^b_{t-1,t}, \quad \text{Select Change Statements} \tag{42}$$

$$\hat{F}^b_{t-1,t} = \sum_c f^{b,c}_{t-1,t} \quad \text{Select Flow Decompositions} \tag{43}$$

The reporting of aggregations from the additional energy dimensions can utilize existing reports to add e.g., additional columns to the statement of financial position / balance sheet. New reports (aggregating and explain mutations of environmental accounts, internal energy stocks etc) provide more focused disclosures. While the selection of what to disclose is open, such integrated energy reporting has the important and non-trivial attribute that whatever *is* disclosed is deeply linked and reconciled with an entities financial activity (Principle 7) 3.5.

3.5 Integrated Accounting Principles

We can now list a number of important principles or design choices that underpin integrated energy accounts:

- **Principle 1: Multi-value Accounts and Transactions following DEB** Accounts and Transactions encoding mutations of accounts are generalized to hold additional measurement values for each economic artifact recognized as financially or energetically significant. The integrated (financial and energy) state of an entity is updated by applying atomic transactions that modify all affected measured or attributed qualities of an account. Accounts and Transactions satisfy double-entry balance constraints for each measured dimension.

- **Principle 2: Alignment of Scope** Financial accounts define the basic perimeter and scope of the integrated accounting system, in particular the asset and liability segmentation is adopted for the attribution of energy values to liabilities. Additional accounts are introduced to ensure the integrity of the extended system as required
- **Principle 3: Holistic and Consistent Measurement** The measurement process defines additional methodologies for quantifying a consistent set of energy qualities (minimally physical and embodied energy) for each one of the recognized accounts.
- **Principle 4: Fair Attribution** Attribution methodologies enforce consistent overall asset and liability balance also when the causal links of assets with liabilities are not obvious.
- **Principle 5: Enforcement of Conservation Laws** Energy conservation laws are expressed by requiring that energy qualities in transactions balance when summed across all relevant energy qualities (stoichiometric balance).
- **Principle 6: Role of Equity** The equity relation retains its role as residual and perpetual liability. In the absence of other liabilities all changes to an entity's stock of energy are attributed to this account.
- **Principle 7: Integrated Reporting** Periodic reports identify accounts of specific interest and explain their start and end-of-period changes. This explain function is constructed by aggregating transactions over classes of similar nature. Reports present the monetary and energy facets of the entity as reconcilable parallel streams.

3.6 Comparing and Contrasting Monetary and Energy Dimensions

The following table aims to provide a summary of similarities, analogies and differences between the financial and energy dimensions of accounts.

Concept	Financial Dimension	Energy Dimension
Accounting Entity	An economic agent that uses accounting to track and report a financial view of its economic state	An economic agent that uses accounting to track and report a physical and embodied energy view of its economic state
Measurement	Determination of verifiable financial value from a menu of agreed methodologies	Determination of verifiable physical and embodied energy values from a menu of agreed methodologies
Value Sign	Positive when owned, negative when owed	Positive when owned, negative when owed
Economic Production	Uses input economic resources to create added financial value. There is no concept of financial waste but there is a concept of value destruction	Uses input physical energy and materials to create added energy value (lower entropy artifacts). Produces waste (higher entropy artifacts) which is captured as part of embodied energy
Assets	Stock accounts of elements that are expected to inject economic value into the entity in the future (they indicate opportunity for value creation)	Stock accounts of elements that are expected to inject energy potential into the entity's inventory (they indicate opportunity for useful work)
Liabilities	Stock accounts of elements that will extract economic value from an entity in the future. They indicate responsibility to external parties to meet liabilities	Stock accounts of elements that get attributed the energy potential of an entity's inventory. They indicate the responsibility of external parties for an entity's energy profile
Equity as Special Contract Liability (Perpetual, Residual Interest)	Is attributed any residual financial value. Concept of Net Worth	Is attributed any residual physical and embodied energy. Concept of Net Energy Liability
Statement of Position	Articulates financial balance across an entity's asset/liability boundary	Articulates energy balances across an entity's asset/liability boundary
Statement of Performance	Explains the change in net worth. Temporary value accounts group transactions by their nature to facilitate explaining	Explains the change in net energy liabilities. Temporary energy accounts group transactions by nature
Double Entry Bookkeeping (DEB)	All transactions and accounts must balance financial value	All transactions and accounts must balance physical and embodied energy value
Conservation Laws	Not applicable	Inflows, outflows, storage and conversion of physical energy to embodied energy must balance across energy qualities
Measurement Identities	Cash has measured value equal to face value	Physical energy is converted to an equal amount of embodied energy
Medium of Exchange	Cash (Money)	Barter (in principle)
Medium of Exchange Accounting	Cash Flow Accounting and Statement	Not applicable
Taxation	Tax Liabilities on Financial Flows	Tax Liabilities on Physical Energy or Embodied Energy (Putative)
Special Accounting Entities	Banking Sector (Money creation)	Energy Sector (Primary energy creation)

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