Open Risk White Paper

energyLedger: Integrated energy accounting using relational databases

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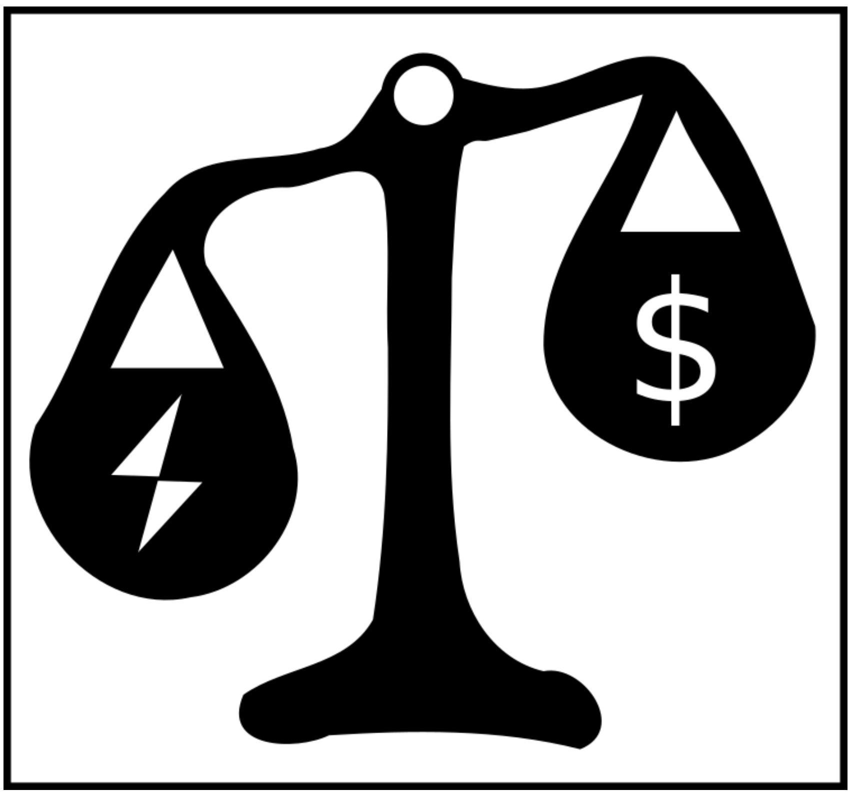

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Abstract

In this Open Risk White Paper we demonstrate a concrete implementation of an integrated energy accounting framework using relational database technologies. The framework enables accounting of non-financial disclosures (such as the physical and embodied energy footprints of economic transactions) while enforcing the familiar double-entry balance constraints used to produce conventional (monetary) accounts and financial statements. In addition, it allows enforcing constraints associated with the flow and transformations of energy that can happen inside the organizational perimeter. We establish the conceptual relation of such an approach with the currently developing European Sustainability Reporting Standards (ESRS) and emissions attributions standards such as PCAF. A simplified sequence capturing transactions of a fictional venture helps illustrate how this type of accounting might work in practice. We also document the details of an open source relational database implementation of the concepts.

Further Resources

- The Open Risk Manual is an open online knowledge base covering diverse domains of risk management.
- The Open Risk Academy offers a range of online courses around risk, portfolio management and sustainable finance, which utilize the latest in interactive eLearning tools. Please inquire at **info@openriskmanagement.com** about eLearning possibilities.
- The Open Risk Commons is a meeting place and primary venue for discussing open source risk models.
- Open Source Risk Repository is our online repository of libraries, tools and frameworks that support quantitative analysis of diverse risk and portfolio management tasks. An SQL schema for energyLedger is available at the dedicated (energyLedger repository).

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1 Motivation and Overview

Integrated energy accounting is an accounting system that describes economic entities (businesses or organizations) and their activities in a manner that captures both the financial side (monetary amounts) and various measures of their *energy footprint* in a consistent and reconciled manner. The broader field of sustainability-linked accounting and reporting sees currently rapid development. It has been widely recognized [1] that documenting the various adverse economic externalities of modern industrialized scale economic activity and adjusting the associated risk perceptions is important to induce environmentally friendly investments and reduce harmful ones. Updated accounting and reporting practices are significant enablers of that transition.

Integrating sustainability related impact factors in models of the economy is well developed at the macro-economic level. It is currently most actively pursued in the construction of aggregated input-output (IO) models[2]. An extension of input-output models to address the energy dimension of economic systems (EEIO) was tackled first in the work of [3], in the context of the energy crisis of the seventies. Currently statistical agencies have built substantial EEIO models that provide comprehensive *physical energy flow accounts* (PEFA). Such models record the flows of energy from the environment to the economy (natural inputs), within the economy (products), and from the economy back to the environment (residuals). Such macro EEIO frameworks already guide national policies and are continuously developed.

In this paper we focus on granular (micro) integrated energy accounting as might be practiced by businesses or organizations. We build on the conceptual models and principles that were introduced in [4], providing detailed examples and implementations. A core aspect of the framework is that it maintains the fundamental double-entry book-keeping architecture of conventional accounting. While it is possible to consider financial and non-financial accounts as more or less *separate systems* and only require reconciliation at very high level. Such approaches may lose conceptual integrity. Historically financial accounting adopted double-entry accounting as an information tool in order to more rigorously *balance* sets of accounts and thus help produce more reliable representations of the true state of an entity. The balancing requirement generalizes naturally to multi-dimensional sets of measured "qualities" and would be more robust for exactly the same reasons. A general multi-dimensional DEB accounting framework has already been discussed [9] in the context of property theory. In [4] we discussed how a multi-dimensional framework can be augmented to enforce the additional accounting constraints of energy systems, and *attribution methods* that bring into the accounting picture the financial liabilities of an entity.

We will link the discussion to the currently unfolding efforts to develop frameworks for sustainability related *reporting* standards [5, 6, 7]. While these efforts are still under development, they are concrete legislative (thus binding) initiatives already in place that provide important context. In particular, CSRD, (Directive (EU) 2022/2464) is already adopted and now creates the conditions for more detailed developments. The CSRD provides an outline of required reporting while actual disclosure requirements are being developed by the technical advisory group EFRAG. In November 2022, EFRAG submitted the draft European Sustainability Reporting Standards (*ESRS*) to the EU Commission for review and a detailed transition roadmap for finalization and adoption is in place. The ESRS standards span the complete range of sustainability related disclosures. In Section 2 we will review and discuss a *subset* of energy related topics to provide current context and motivation for integrated energy accounting platforms.

In Section 3 we will attempt to remove some of the abstraction associated with such new accounting considerations and illustrate with a concrete exercise. We will track the activities of a fictitious company *EcoWidgetCo* (EWC), an ambitious new green venture that builds micro-mobility widgets using recycled materials and (mostly) renewable energy. We will sketch how several periods of EWC's economic operations could be represented in an integrated energy accounting context.

Finally in Section 4 we will make the concepts and ideas of granular integrated energy accounting more concrete by presenting a technical implementation using the tools of relational databases. This illustrates the practical feasibility using currently available tools. The implementation is open source and available in a dedicated repository [8].

1.1 Designing Integrated Energy Accounting - Key Dimensions

In principle integrated energy accounting can be considered as an *open-ended super-set* of conventional financial accounting. The "open-ended" part of this sentence refers to the fact that the scope and detail of energy related elements that are accounted for and disclosed is ultimately a question of utility and cost-benefit analyses of various stakeholders. This is in complete analogy with the evolution and shape of financial accounting and reporting which is highly idiosyncratic in which aspects of a business entity it documents. It is to be expected that *internal management objectives* will in general be more demanding than externally required disclosures. Yet external stakeholders may have other requirements: E.g., the detail to which they require to piece so-called *whom-to-whom* relations which can variably be seen in information requirements such as Value Chain Transparency, Scope 3 Emissions etc. In accounting parlance these would be forms of so-called *quadruple-entry accounting* which reconciles accounts across multiple distinct entities.

The "super-set" part of the first sentence expresses a design choice around integrated energy accounting, namely that it *encompasses* purely financial accounts and these are not affected by the presence or not of the energy accounting. The opposite is not true: accounting and reporting on the energy profile of economic activity is intimately linked to the financial accounting representation of an organization. For example the *organizational boundary* is captured primarily by its legally and contractually recognized financial relations. This primacy of financial accounting in defining the organizational boundary is not an immutable law of nature, but follows from being for the longest time the primary approach to documenting economic activity.

In Fig. 1.1 we provide a visual overview of the different elements of the integrated energy accounting system. It illustrates the multitude of non-trivial choices that are inherent in designing such systems. The next list briefly introduces and discusses some important ones.

- the *scope of accounting* and reporting entities (the type of different organizations that adopt the accounting system).
- the *organizational perimeter* of reporting entities, namely what is considered part of the organization.
- whether accounting covers and reconciles *assets and liabilities*. Double-entry accounting systems naturally introduce balance equations and balance sheets. Yet liabilities are financial contracts without intrinsic energy footprint, if they are to be included suitable attributions methods must be devised.
- which forms of *physical energy* are tracked. This is primarily a question of managing granularity and availability of valid conversion rates
- whether the system tracks *embodied energy*. Embodied energy is a virtual construct that help track energy consistently across disperse value chains.
- how extensive and granular the enforcement of *consistency and integrity* between an entity's financial and energy accounts.
- the type of *reports* that can be generated. Financial reporting has culminated over centuries in a number of practices (standard statements etc.). The corresponding structure of sustainability reports is currently still under development.

1.2 Representations of Accounting Systems

There are various ways to represent (define and describe) the design of a practical integrated energy accounting and reporting system. The traditional (and official) approach comprises long textual descriptions embedded in formal documentation. While definition, it has disadvantages in that it makes less visible the core relationships between various accounting concepts. A *mathematical representation* is, instead, a concise representation that is essentially a small list of equations and associated abstract variables. Yet mathematical formalism is seldom encountered in accounting literature except occasionally in research/academic context [10]. The mathematical presentation of integrated energy accounting as we envisage it here was given in [4] and in the interest of brevity it will not be repeated here. The interested reader may find it useful to review.

Another relevant representation approach concerns *information technical aspects* of accounting systems, namely implementations in computer systems. Such descriptions can be developed using e.g., UML diagrams to outline the relevant data objects and their relations and/or code (for instance SQL code) that illustrates the concrete realizations of relational objects. In section 4 we will go over the details of a (very minimal) implementation of an integrated accounting ledger. The advantage of technical representation is that it makes concrete the mathematical abstractions and creates placeholders that can illustrate typical information flows.

While textual, mathematical and technical descriptions provide essential information, a detailed numerical worked-out, a step-by-step example of different transactions may help bring these concepts to life. This is what we provide in section 3 where we will follow a hypothetical company through a number of transactions that span a range of interesting phenomena relevant to integrated energy accounting.

1.3 Energy Measurement Dimensions

Energy is a quality and a quantity that is defined for any physical system and is one of the most fundamental notions in all of physics, irrespective of which spatial scale, material composition or nature of physical forces we might consider. The use of energy *measures* in sustainability accounting context is but a tiny subset, comprising specific forms of energy that are considered high-impact and relevant. *Primary energy* is physical energy as first extracted from the natural world and brought into the economic system (as fossil fuel energy, solar energy, wind power etc). textitSecondary energy is physical

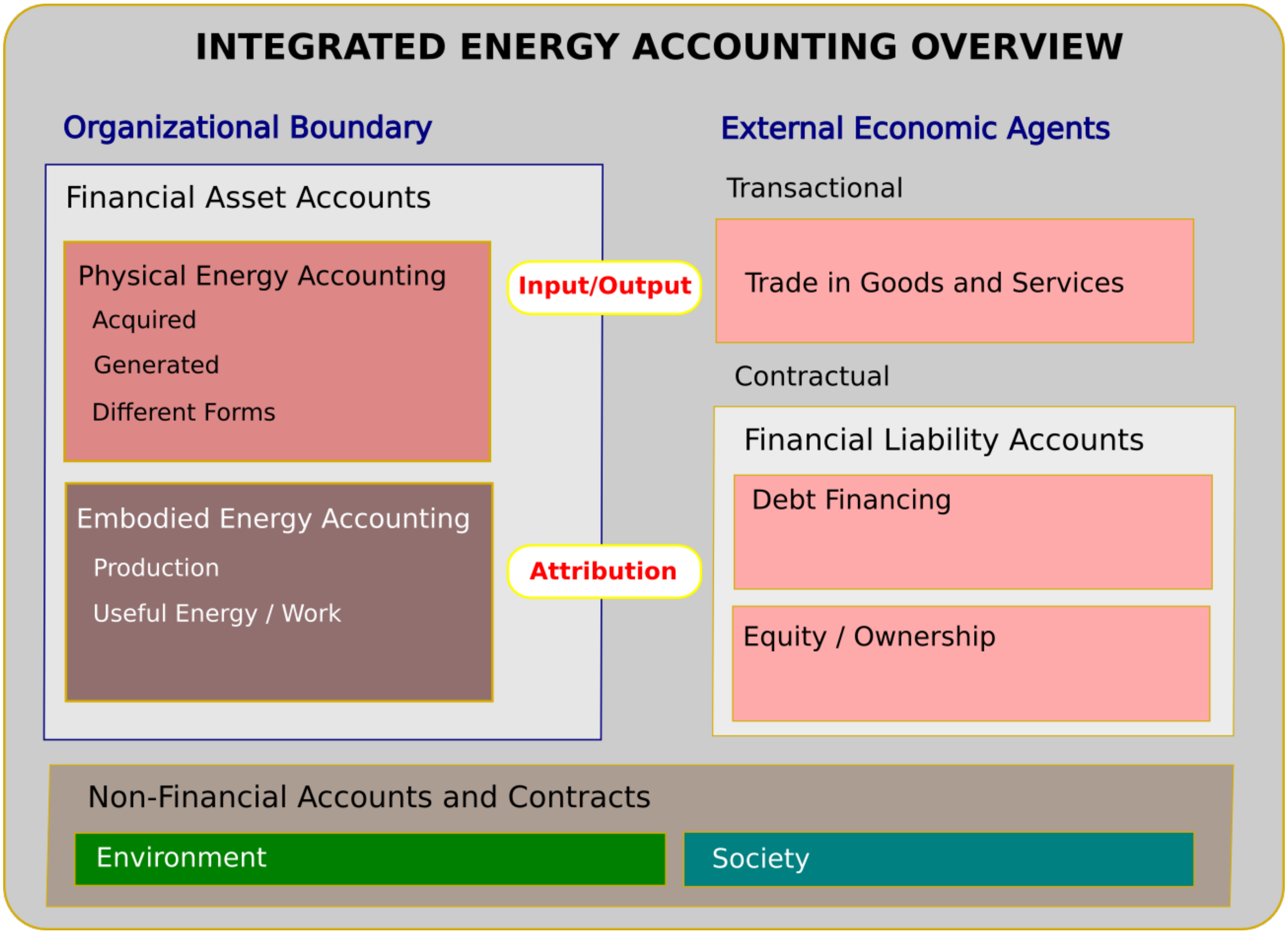

Figure 1: A schematic overview of the different elements that an integrated energy accounting system must (in-principle) consider. The organizational boundary of a reporting entity is maybe the key element to consider first. It is something defined legally in terms of ownership and, by-and-large, it is the same as the financial reporting boundary.

At the most basic level energy accounting would focus on the asset side of the financial balance sheet and examine its physical energy profile (for example the rate of acquisition or generation of energy for business purposes). Reporting these **energy mix metrics**, along with the corresponding **energy intensity ratios** using monetary flows from the financial reporting side comprises the essence of current reporting requirements as expressed e.g., by the ESRS Standards.

Linking the energy consumption activity of an entity with its *upstream and downstream value chains* leads to hybrid environmentally extended input-output models which are currently available only at the macro-economic level, e.g. the EU PEFA system.

Connecting the energy profile of the entity with financial counterparties that provide different forms of capital leads to current environmental footprint attributions models pursued by the financial industry such as PCAF. While material use of physical energy must in any case be accounted for, the embodied energy in various goods or services offers the most coherent representation as it ties business processes that may be disconnected in time and space yet serve the same economic purpose: providing a final product.

Last but not least, an entity may have non-trivial interactions that are not captured under financial exchanges with economic agents. This segment includes the environmental "account" that may serve both as a source of high-quality energy and as a sink of waste energy. It also includes broader contracts with society, for example the indirect impacts of energy infrastructure on communities.

energy that has been transformed to empower alternative economic uses (e.g., distribution via electricity networks). There is already an energy loss in its transformation process from primary energy. Physical energy may be *Self-Produced* or *Acquired Energy* (when an entity receives energy from a third party). The distinction is important in the context of establishing the provenance of energy flows. *Useful energy* is physical energy that is available to end-users to satisfy their energy needs. It is also referred to as *energy services demand* [12].

In a sustainability accounting context there is the important concept of *embodied energy* of a product or service. This is the *historical aggregation* of all measured physical energy transformations that have enabled its production process. Herendeen and collaborators [3] first developed a theory of embodied energy in which the *energy cost* of a product is obtained by accounting for all these energy flows. Embodied energy is essentially an accounting information artifact (it does not exist in nature as such). It recognizes that spatially and temporally dis-aggregated economic processes are causally connected from an economic perspective.

Finally, again important for sustainability, there is a clear dichotomy between non-renewable and renewable forms of primary energy. *Renewable energy* is energy taken from sources that are inexhaustible. As such, renewable energy covers wind, solar (solar thermal and solar photovoltaic) and geothermal energy, ambient energy, tide, wave and other ocean energy, hydropower, biomass, landfill gas, sewage treatment plant gas, and biogas. *Non-renewable energy* is, in contrast, energy which cannot be identified as being derived from renewable sources. Fossil fuels such as oil, natural gas, and coal are examples of non-renewable resources. The *renewability* attribute of an energy type is ultimately a legal fact (e.g., Directive (EU) 2018/2001).

What unifies all the above notions of energy is that they can be expressed in common measurement units. E.g. *MWh* or other units (e.g. tonne of oil equivalent or *toe*) that have a fixed and immutable relation with each-other (unlike currency exchange rates). A set of measured energy dimensions that covers current sustainability accounting approaches at both the macro and micro-level would comprise (schematically):

- Physical Energy *Species*
  - Energy Type 1 (Non-Renewable)
  - Energy Type 2 (Non-Renewable)
  - ...
  - Energy Type N (Renewable)
- Embodied Energy Species

where Embodied Energy, being a derived measure can also be decomposed along the same physical energy mix. The simplest integrated energy accounting system will involve only two energy species: physical energy (tangible energy that can e.g., be measured in the real world with calorimeters) and embodied energy which is an abstract quality attached to goods and services to track the physical energy that has been involved in their production. An even simpler system only involve a single species of physical energy. The challenge for a system that does not track embodied energy is that it miss relevant linkages of economic activity. For example, around 27.2% of the world's CO2 emissions from fuel combustion in 2015 is linked with international trade. [13]

1.4 Measurement and Attribution of Energy to Assets and Liabilities

In standard accounting, measurement (also termed valuation) establishes the monetary value of all relevant economic artefacts such as physical assets or financial contracts). This procedure is fully decoupled from the accounting "database" itself but is part of the overall accounting standards and permissible methodologies. In practice any item accounted-for that does not have a readily associated *face value* requires the use of some measurement methodology.

The measurement situation is not different in integrated accounting where energy values must be obtained according to established methodologies. One area where there is significant deviation is the attribution of energy values to *liabilities*. In financial accounts the concept of attribution shows up only in the measurement of *equity*. Accounting equity gets attributed its numerical value on the basis of the value of the remaining assets and liabilities. Thus not with reference to market value or other direct measurement methods. Other liabilities *can* be measured directly and individually on the basis of various methodologies.

In the context of integrated energy accounting, attributing energy metrics to liabilities becomes a major methodological component. In a nutshell, as with the monetary value of accounting equity, the attributed energy value of liabilities cannot (in general) be measured directly and individually. For example, a general purpose loan can be valued on its own but we cannot attribute its "fair" energy contribution that matches the asset side of the balance sheet, without reference to the other assets and liabilities. Various alternatives are possible hence the accounting system must be flexible to accommodate all plausible approaches

It is of-course possible *not to balance the energy books* across assets and liabilties, but limit the accounting and reporting on the asset side of the balance sheet. While maybe offering an expedient starting point, the long experience of accounting history suggests that a consistent scheme that reconciles within (and ultimately across) entities is an important feature to help preserve the integrity of accounts.

The monetary measurement of assets and liabilities leads to the standard *balance sheet equation* which equates assets, liabilities (including equity) at all times. This is implemented as a set of conventional signs (plus or minus) that prescribe how the values of different accounts are to be added across the balance equation. The generalization to integrated accounting is (in principle) straightforward: The measurement of assets and liabilities must balance along all the energy measurement dimensions that we might chose to account for.

At any given period one might ask what is the balance of the complete set of accounts of the accounting entity. This is a matrix of numerical values. After a number of periods of economic activity and transactions might look like as (we'll see a worked out example in Section 3) (where the first row indicates the account and subsequent rows measured values):

$$V = \left[\begin{array}{cccccccc} C & F & S & M & I & \mathbf{P} & \mathbf{L} & \mathbf{K} \\ \hline 120 & 1000 & 300 & 150 & 600 & 0 & 0 & 2170 \\ 0 & 0 & 500 & 0 & 0 & 0 & 0 & 500 \\ 0 & 100 & 0 & 600 & 700 & 0 & 0 & 1400 \end{array}\right] \tag{1}$$

While in standard accounting the balance equation will only involve the first row (the monetary aspect), more general systems require as many balance equations as the range of measured qualities. In analogy with the balance equation of standard financial accounting which is effectively the sum of a vector (array) of values, our accounting matrices having their *row sums equal to zero* will be called *balance matrices*. Balance matrices are the mathematical foundation for an integrated accounting system as both accounts and transactions are expressed using such matrices. In the case of energy accounting further constraints on the possible shapes of balance matrices are placed by *physical properties and conservation laws* as we will discuss below.

1.5 Transaction Types

Over time the entity's inventory of economic resources and relations changes and this must be accounted for. E.g., an entity's claims of ownership evolves due to operating, financing and investment activities materialized either immediately with cash exchanges or via longer term contracts. The classic transaction concept can be readily generalized to handle dual or higher dimensional measurements as required to keep track of energy qualities. In order to satisfy the design of double-entry bookkeeping, accounting transactions must themselves satisfy balance conditions for all measured qualities. Mathematically both accounts *and* transactions are *balance matrices* with their row-wide elements always summing up to zero. Each transaction is a similar multi-dimensional array of values, where one dimension ranges over the accounts and the other ranges over the measured dimensions (qualities). The values of this array (*the transaction leg data*) are derived following similar methodologies as per the measurement of underlying accounts.

Given a matrix of accounts as per above, all transactions (mutations) in the accounting system are ultimately represented by flow (change) matrices that are added to the accounts, element-by-element, using conventional addition and subtraction. A concrete transaction changing the cash and equity accounts might be depicted as follows (we'll see and discuss more examples later).

$$T = \left[\begin{array}{cccccccc} \Delta C & \Delta F & \Delta S & \Delta M & \Delta I & \Delta\mathbf{P} & \Delta\mathbf{L} & \Delta\mathbf{K} \\ \hline 120 & 0 & 0 & 0 & 0 & 0 & 0 & 120 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \tag{2}$$

Boldface accounts concern *changes to liabilities* and follow the negative sign convention. The balance requirement for transactions is expressed exactly as for accounts (the rows must sum to zero). These balance constraints apply *separately* for each horizontal (row) dimension. Given balanced transactions, when these are applied (posted or added to the accounts) they produce balanced accounts.

There is a vast array of possible transactions. We might distinguish internal versus external transactions (that involve other economic parties). Internal transactions might capture real activity (production or consumption) by adjusting internal accounts of stocks and inventories or be purely bookkeeping operations. External transactions with other economic entities might be cash or credit based. Abstract transactions with environmental accounts can represent non-monetary extraction of resources. Many transactions are purely contractual in nature (financing, investments or distributions to equity).

All of the above distinct types of transactions affect the different measured values (financial, physical energy, embodied energy) as encoded in specific transaction values. Some important transaction categories that highlight the different ways qualities of energy might percolate through an entity's accounting system:

- *Trading Activity Transactions*: This is the typical economic exchange of buying and selling goods or services. In principle all purchasing or selling of goods and services from other economic entities involves exchange of some physical and/or embodied energy. E.g., any material goods transaction changes the stock of embodied energy (which must be reflected on both asset and liability sides). Purchasing or selling physical energy changes accounts in a similar way. Accounting for physical energy might need the introduction of dedicated accounts that reflect the storage of physical energy in fuel containers or other forms of energy storage. Economic Input-Output models of the aggregate economy establish the large scale trading patterns between sectors.

- *Production Activity Transactions*: These transactions are purely internal to organizations and are not visible in standard reporting. They express the transformation of physical energy into embodied energy in the process of production. Any production activity, whether captured explicitly as an inventory increasing transaction or implicitly via income / expenses transactions will affect some physical energy accounts with a decrease and the embodied energy account with an increase. Such transactions are special as they must satisfy physical conservation laws in addition to the accounting balance equations.

- *Financial Activity Transactions*: Pure financial activities (usually contractual activities) such as equity investments, general purpose financing or various financial hedging contracts do not involve directly measured amounts of physical or embodied energy but may change the attribution of physical and embodied energy to accounts.

- *Energy Production Transactions*: This is special type of transaction only relevant for entities that can produce energy.

The above categorizations are just indicative and adapted to energy considerations as opposed to more general sustainability accounting. In any case we assume that there is a complete taxonomy and all relevant activity is captured with such transactions.

1.6 Attribution to Liabilities

The question of *attribution* of impact or footprint is fundamental in all sustainability discussions and frameworks. It reflects the obvious fact that the economy is a complex network of interdependent agents. Hence establishing a factual basis as to who is *responsible* or benefiting from particular adverse impacts is not entirely straightforward. In our current context the question of attribution emerges from the balance requirement applied to non-monetary alongside monetary measures. In presence of multiple liabilities there is a methodological question of how to balance (attribute) the asset energy measurements on the liability side. Different approaches are conceivable with different implied philosophies:

- Attributing total energy to equity irrespective of other liabilities. This is a simple approach but may create distorted incentives.

- Residual attribution to equity *after* accounting for liabilities in cases where the use of financing proceeds is known and can be tracked.

- Attributing energy also to generic liabilities pro-rata some monetary metric.

There are many possible variations and fine-tuning of the above directions. E.g., one might argue that the longer the maturity of liabilities the more enabling and thus *responsible* for the footprint of the entity. An important practical and conceptual implication of using such attribution schedules that take into account the presence of multiple generic liabilities is that attribution factors have to be *recomputed* at each point-in-time for each new transaction. This may create a certain *path-dependency* in the evolving attribution profile, something that is not present in conventional accounting where attribution is limited to establishing the residual value of accounting equity.

1.7 Representing Energy and Entropy Laws

Both the standard balance sheet equation and the generalized balance across energy values we have discussed above have the general form of *charge conservation laws*. Such laws are governing the nature of flows in and out from nodes in a network. This is in analogy with *Kirchhoff's first law* that is governing the conservation of electrical charge that flows in and out of the nodes in an electric circuit form. In our context all assets (internal accounts) are incident edges into a node (which represents the entity) and all liabilities (external accounts) are edges emanating from the node. The same picture applies both statically (the absolute values of assets and liabilities) and dynamically (the deltas introduced by transactions).

Articulating an energy conservation law relies on a suitable definition of the system's perimeter. An accounting entity is not a closed system in either the physical or the embodied energy sense. Various actual energy forms flow in and out of the organizational perimeter as that is defined through legal ownership claims. Similarly material artifacts flow in and out as well, in a constant exchange of embodied energy. The external environment might be represented by a set of other economic entities and/or environmental accounts. The physical properties of energy are partially expressed by the above conservation law. When a quantum of energy (of any type) enters the organizational perimeter (associated with an asset) it must automatically be balanced by external accounts that are assigned the provision of that energy unit. This expresses energy conservation across the organizational boundary.

In the absence of energy transformations between various energy types energy values are conserved separately (in a horizontal sense, the sum of assets matching the sum of liabilities) and there is no need for any other constraint. Yet energy transformations *between* energy qualities are possible (typically in production activities). The law of energy conservation within an entity's perimeter sets a constraint that transactions representing such transformations must preserve total energy on the asset side of the entity. This automatically preserves total energy also on the liability side. This type of constraint is analogous to *stoichiometric balance equations* in chemistry.

The above two types of conservation laws jointly express the first law of thermodynamics (energy conservation). The first ensures energy accounting is valid when any type of energy enters or leaves the entity perimeter, whereas the second ensures that internal processes also preserve total energy.

Finally, a more subtle but physically equally relevant law concerns another thermodynamic concept, *entropy*. Without going into physics details, this dimension concerns the ability of the physical energy forms available to the organization to produce work. Notice how the energy conservation laws we discussed above do not prescribe the direction of energy flows. For example embodied energy could be converted to physical energy provided the balance is kept yet this is not physically allowed: Products and Services cannot be un-bundled to fully recover the original amount and type of energy. Production always involves energy losses to the environment that are in general non-reversible.

To explicitly express this one-directional flow of entropy we need to focus on transactions that capture *pure energy conversions* from one energy form to another. The second law of thermodynamics requires that the entropy of an isolated system that undergoes transformation does not decrease. In practice the implication for integrated energy accounting is that (in isolation), thus for internal transactions the embodied energy inside an organizational perimeter can only increase.

2 European Sustainability Reporting Requirements

In this section we briefly review the energy-related reporting requirements of the newly introduced (but not yet final) European Sustainability Reporting Standards (ESRS). While the ESRS standard concerns specifically European businesses sectors (and non-European entities with activity in Europe) they reflect a global drive towards sustainability linked energy disclosures. At the highest level, the United Nation's Sustainable Development Goals (7) promote affordable and clean energy. They include targets to increase substantially by 2030 the share of renewable energy in the global energy mix. In that respect, accounting and reporting regulation globally will in principle induce businesses to disclose information on their energy consumption and mix, including a dis-aggregation of their total energy consumption from non-renewable and renewable sources.

The ESRS standards, being an early and well developed set of requirements, help serve as a blueprint as to what are deemed essential sustainability related energy disclosures. While the framework of this paper is not aiming to be a detailed implementation of the ESRS standards the discussion establishes important conceptual linkages and places the sustainability disclosure proposals within the broader information flows inside and across economic entities.

A specific ESRS Standard (E1)[14] covers disclosure requirements related to the following sustainability matters: *Climate change mitigation*, *Climate change adaptation* and *Energy*. Energy reporting is thus part of cluster of related disclosures. ESRS states that organizations should consider including metrics on climate-related risks associated with water, energy, land use, and waste management where relevant and applicable. Energy accounting is very closely related to GHG emissions management given that the largest segment of GHG emissions are energy related. ESRS requires, for example, disclosures around renewable energy deployment and energy efficiency, electrification, fuel switching but also phase-outs or substitution of products and processes and supply-chain decarbonisation.

Under ESRS, entities must disclose the policies they have adopted to manage material impacts, risks and opportunities related to climate change mitigation and adaptation. Specifically the ESRS requires that reporting entities indicate whether and how their policies address energy efficiency and renewable energy deployment. In this respect integrated accounting provides data points and metrics for making policies and forward looking statements more observable and credible.

While the broader sustainability transition is *coupled* with the energy transition, it is not identical to it. This is clearly seen in the distinct ESRS pillars or sections that cover the diverse environmental, social and governance factors (ESG Risks). On the other hand energy accounting links deeper to an entity's management and internal accounting. While GHG emissions are a negative externality that must be contained, energy is (still) an essential and scarce resource and helping understand and manage the energy profile of a business would be useful even in the absence of environmental impacts. Integrated energy accounting can thus help provide a well defined and objective basis for both internal management and external reporting.

2.1 Materiality of Energy Accounting

While even the tiniest bit of economic activity has an energy footprint, not all of it can or should be accounted for. The example of financial accounting is instructive in this sense: even after centuries of development a large segment of economic activity is informal and not accounted for directly. There is an implied cost-benefit analysis that determines which subset will be formally accounted for in a given context. The implication is that, seen holistically, the accounts will not balance but will exhibit some *leakage*.

The scope and required detailed of integrated energy accounting may evolve as both sides of the cost-benefit analysis evolve (e.g. reduced costs from more effective use of digital tools and additional benefits from more accurate management of the energy footprint). For our present purposes it suffices to align to the materiality profile endorsed by major sustainability accounting initiatives. For example, EU legislation introduced the concept of *high climate impact business sectors*. High climate impact sectors are those listed in NACE Sections A to H and Section L:

- NACE Section A - Agriculture, Forestry and Fishing
- NACE Section B - Mining and Quarrying
- NACE Section C - Manufacturing
- NACE Section D - Electricity, Gas, Steam and Air Conditioning Supply
- NACE Section E - Water Supply, Sewerage, Waste Management and Remediation Activities
- NACE Section F - Construction
- NACE Section G - Wholesale and Retail Trade
- NACE Section H - Transporting and Storage
- NACE Section L - Real Estate Activities

Integrated energy accounting would be have most impact when adopted by entities that are active in these business sectors.

2.2 Disclosure Requirement E1-1: Transition plan for climate change mitigation

A general thrust of the ESRS is that businesses must disclose *transition plans* for climate change mitigation. Stakeholders must be able to understand the business's past, current, and future mitigation efforts to ensure that strategy and business model are compatible with the transition to a sustainable economy (limiting global warming and achieving climate neutrality). This includes exposure to coal and oil and gas-related activities.

The majority of the disclosed transition plan elements must address directly *GHG emission reduction plans and targets*. The role of energy mix and energy intensity in this context is indirect: the energy profile of a business is intimately linked but does not exclusively determine its GHG profile (it is not equivalent to it). Nevertheless the overlap is sufficiently important that under paragraph 15 (d) of ESRS E1, "an entity must disclose a qualitative assessment of the potential locked-in GHG emissions from the undertaking's key assets and products. This shall include an explanation of if and how these emissions may jeopardize the achievement of the undertaking's GHG emission reduction targets and drive transition risk, and if applicable, an explanation of the undertaking's plans to manage its GHG-intensive and **energy-intensive assets and products**.

We should stress that an accounting framework only partially overlaps with the needs of qualitative sustainability disclosures. E.g. it is not particularly suited for the analysis and presentation of business models, policies and/or for tracking plans and targets. Yet the E1-1 disclosure requirements do suggest that an accounting framework design that helps identify and group (using a suitably organized chart of accounts) the energy-intensive assets and products (goods or services) that entity owns or provides will aid the process of verifying its transition path.

2.3 Disclosure Requirement E1-5: A. Information on energy consumption and mix

Disclosure Requirement E1-5 concerns energy consumption and mix and is thus central from an energy accounting perspective. It requires (ESRS E1 paragraph 33 / 34) reporting entities to provide information that will help understand:

- the total energy consumption in absolute values
- the improvement in energy efficiency
- the exposure to coal, oil and gas-related activities
- the share of renewable energy in the overall energy mix

ESRS requires a reporting period for the sustainability report that is consistent with the one adopted for financial statements. This consistency is essential, for example, when deriving hybrid ratios that use information from both reporting sides, such as the energy intensity ratio. In an integrated energy accounting framework this requirement is satisfied automatically at a granular level (rolling up a set of transactions over an specified period will aggregate monetary and energy figures consistently.) Energy consumption from non-renewable sources must be dis-aggregated by sources (only applicable to high climate impact sectors). These disclosures must include the total energy consumption in Mega-Watt-hours (MWh) related to own operations as follows:

- total energy consumption from non-renewable sources for high climate impact sectors dis-aggregated by:
  - fuel consumption from coal and coal products
  - fuel consumption from crude oil and petroleum products
  - fuel consumption from natural gas
  - fuel consumption from other non-renewable sources
  - consumption from nuclear products
  - consumption of purchased or acquired electricity, heat, steam, and cooling from non-renewable sources
- total energy consumption from renewable sources dis-aggregated by:
  - fuel consumption for renewable sources (including biomass, biogas, non-fossil fuel waste, hydrogen from renewable sources, etc.)
  - consumption of purchased or acquired electricity, heat, steam, and cooling from renewable sources; and
  - consumption of self-generated non-fuel renewable energy.
- dis-aggregate and disclose separately non-renewable energy production and renewable energy production in MWh

Note that the focus of the required reporting is on *physical energy* and there is no requirement to account for the embodied energy of *upstream or downstream* economic exchanges. Both here and in the energy intensity disclosure discussed next the identification of the business sector under which an economic activity creates revenue and consumes energy is an important consideration. In the ESRS the NACE Sectoral Classification of Economic Activity defines the perimeter for both energy consumption and revenue that must be matched. As a general requirement the accounting system must be able to classify and group transactions by business sector.

In terms of measurement units, the ESRS requires that all quantitative energy-related information is reported in either MWh in Lower Heating Value or net calorific value. If raw data of energy-related information is only available in energy units other than MWh, such as Giga-Joules (GJ) or British Thermal Units (Btu) or volume / mass units, such as cubic feet or gallons, kilograms (kg) or pounds (lb) it must be converted to MWh using suitable conversion factors. Conversion of energy units between accounts is somewhat analogous to the use of different monetary units but simpler in that the exchange rates are fixed.

All quantitative energy-related information is reported as *final energy consumption* (referring to the amount of energy the business actually consumes in the pursuit of its activities). When disclosing self-generated energy consumption (from either a non-renewable or renewable fuel source) the energy is only to be counted once. Self-generated energy that is sold to and used by a third party must be accounted for. Energy that is sourced from within the organizational boundary must not be accounted under purchased or acquired energy whereas steam, heat or cooling received as waste energy from a third party's industrial processes must be under purchased or acquired energy.

The business can only label energy consumption as deriving from renewable sources if the origin of the purchased energy is clearly defined in the contractual arrangements with its suppliers (renewable power purchasing agreement, standardized green electricity tariff, market instruments like Guarantee of Origin from renewable sources in Europe or similar instruments like Renewable Energy Certificates in the US and Canada, etc.).

2.4 Disclosure Requirement E1-5: B. Energy intensity based on net revenue

Financial accounting and reporting typically concerns *absolute monetary values*. In practice an auxiliary system of ratios based on absolute values is heavily used as it helps elucidate the relative weight and role of various economic activities both within an entity and when comparing across entities. The same pattern will be of typical use in integrated energy accounting, with a generalization to three types of ratios:

- Financial Ratios
- Energy Ratios (e.g., share of energy mix include renewable / non-renewable attribute)
- Hybrid Ratios (e.g. energy use per unit of revenue)

Accounting ratios are not, strictly speaking part of the accounting system, but rather post-processing activities. Yet the derivation of consistent hybrid ratios (consistency in the scope of numerator and denominator values) is something that is easier accomplished (at any desired granularity level) in an integrated energy accounting system versus two separate systems that are only reconciled at some level of aggregation.

A second important disclosure in the E1-5 cluster is the energy intensity associated with activities in high climate impact sectors (paragraphs 37 to 40 of ESRS E1). Entities must provide information on the **energy intensity** (total energy consumption per net revenue). The energy intensity disclosure is to be derived from the total energy consumption and net revenue from activities in high climate impact sectors. These sectors must be specified explicitly. From an accounting perspective this requirement becomes non-trivial when the entity is active in multiple business sectors, while only some of them belong to the high-impact class. In such circumstances the overall accounting system must recognize internal boundaries and appropriately classify all economic transactions accordingly.

The energy intensity ratio (EIR) is calculated using the following formula:

$$\text{EIR} = \frac{E}{I} \tag{3}$$

In words, the total physical energy consumption E from activities in high climate impact sectors (in MWh units) over the net revenue I from activities in high climate impact sectors (in Monetary units, e.g., Euros). The denominator (net revenue amount from activities in high climate impact sectors) being a financial measure, the reporting entity must disclose the reconciliation to the relevant line item or notes in the financial statements. In an integrated energy accounting context that reconciliation is greatly facilitated and can happen at arbitrary granularity (energy intensity calculated and attributed to various subsets of economic activity).

As per ESRS guidance, the reconciliation of net revenue from activities in high climate impact sectors to the relevant financial statements line item or disclosure (as required by paragraph 40) may be presented either by a cross-reference to the related line item or disclosure in the financial statements or if the net revenue cannot be directly cross-referenced to a line item or disclosure in the financial statements, by a quantitative reconciliation using a recommended tabular format.

2.4.1 The role of Financial Intermediaries

A business sector that is not considered having a *direct* high climate impact profile are financial intermediaries: Banks, Insurers, Asset Owners, Asset Managers etc. Nevertheless it is recognized this sector has significant *indirect* impact. Namely its investment and risk management policies and practices play a significant role in financing, thus *facilitating* high impact sectors.

The sustainability related disclosures of financial intermediaries thus become an essential element in the overall project of providing transparency into the sustainability profile of economic activities. In fact legislative initiatives such as the EU Regulation 2019/2088 on sustainability-related disclosures in the financial services sector (SFDR) are creating *demand for disclosures* from entities active in high impact sectors. As noted explicitly in [11], *In order to create the necessary data infrastructure for financial market participants to be able to fulfil their SFDR obligations, the draft ESRS mandate the disclosure of all these indicators in all circumstances, irrespective of the outcome of the materiality assessment.*

3 Worked Example: Integrated Energy Accounting at EcoWidgetCo

In this section we will track the activities of a fictitious company *EcoWidgetCo* (EWC), an ambitious new green venture that builds micro-mobility widgets using recycled materials and (mostly) renewable energy. While having genuine sustainability ambitions, EWC is a standard, for-profit, limited liability corporation. It must thus keep proper accounts, produce financial reports and fairly soon, sustainability reports. EWC decides to *augment* its core financial accounting and reporting to offer a complete and consistent picture of not just its monetary condition (solvency, profitability etc), but also its energy profile, both the actual physical energy used and the embodied energy of the economic activities involved in its business model. We will sketch here how several periods of EWC's existence and operations could be represented in an integrated energy accounting context. We will do this by constructing a concrete (if simplified) *charter of accounts* and expressing a number of representative *transactions*. We will not generate integrated financial and energy reports that generalize current reporting conventions (statement of financial position, income statement etc), these can be built as straightforward extensions. At each step we will illustrate how the expanded accounting state (the integrated view of the balance sheet) gets modified when accounting for the evolving energy profile. In the accompanying SQL implementation in Section. 4, this sequence of the sample transactions is included verbatim.

Generally speaking, the range and type of accounts (the chart of accounts) and transactions that must be supported depends heavily on the business model of the accounting entity. Integrated energy accounting, just like financial accounting is likely to require diverse taxonomies and specialized reports adapted to the classes of entities pursuing such accounting and in particular their business model and corporate structure. High impact sectors where energy use is a core aspect of the business model will require more nuanced accounting.

3.1 Business Model

EWC's economic objective is produce and sell mobility widgets at monetary profit. Its integrated accounting system aims to capture in detail the physical and embodied energy flows that underpin its business model. EWC desires that its accounting system properly attributes its energy footprint to the various external liabilities it incurs in the course of business (and in particular its shareholders).

The business model of EcoWidgetCo is, broadly speaking, just the standard model of manufacturing companies. It goes roughly as follows: A widget is assembled from a collection of raw materials by a combination of machines (robots) and human labor and supervision. This assembly process requires the use of physical energy that performs useful work (mostly mechanical and chemical processes). This work puts, for example, the widget elements together and/or modifies their properties. The end result is a lower entropy construct (a functioning and useful widget) that is also economically of added value. During the assembly process beside energy converted to useful work (useful energy) there is also energy loss to the environment (as heat). The total of energy used or lost becomes the embodied energy associated with the widget.

To increase its sustainability profile EWC will procure and use an installation of solar panels that produces high quality energy with reduced environmental impact. Additional energy acquisition may be required from the local electric grid to accommodate manufacturing schedules. The required raw materials are bought in a variety of markets. EWC doesn't have much leverage over their supply chain. Yet they do want to properly account for the footprint of all utilized resources. Finally, widgets are bought or sold from/into the economy in exchange for cash (but also possibly on credit).

3.2 EWC Chart of Accounts

EWC will over time build a large inventory of economically relevant artefacts. This will include rights and claims on physical assets it acquires but also various financial contracts claims or obligations with external parties). Mapping that inventory into accounts in a non-trivial exercise. Given that EWC must keep conventional financial accounts and produce financial reports the initial scaffolding for the chart of accounts can based on the existing body of practice. A mature integrated accounting framework is likely to require additional fine-grained accounts and taxonomies that provide the required resolution. For our purposes the following minimal list of accounts will do:

Integrated T-Account					
Dr			Cr		
CU	EU	EU	CU	EU	EU
10				100	

Table 1: Representation of a generalized T-account where the monetary dimension is augmented by two measured energy qualities. All transactions on accounts generalize naturally but there are some new concepts, e.g. transformations between energy types can capture in more detail the physical footprint of the economic activities of an entity.

Code	Symbol	Description
A01	C	Cash - Measured quality is monetary value (face value)
A02	F	Solar Panel Array and Widget Factory - Measured qualities are value and embodied energy
A03	S	Energy Stock - Measured qualities are value and physical energy
A04	M	Raw Materials - Measured qualities are value and embodied energy
A05	I	Widget Inventory - Measured qualities are value and embodied energy
A06	**P**	Accounts Payable - Measured qualities are value and attributed energy
A07	**L**	Bank Loan - Measured qualities are value and attributed energy
A08	**K**	Equity - Measured qualities are attributed (residual) value and energy

In principle all the above accounts have three associated measurements (expressed in their respective units, currency or energy unit CU, EU). The iconic representation of double-entry accounting in the paper era is the T-Account or T-Chart. While we will not have much use for this visual representation here, it is maybe helpful, in particular for those who might already be familiar with them to represent "generalized" T-Accounts 3.2.

In the above chart of accounts, accounts A01 - A05 would be classified as financial assets while A06 - A08 would be classified as financial liabilities (indicated by boldface). A01 will have measured value equal to its face value and zero embodied energy given that A01 (cash) is a monetary artifact with immaterial energy footprint. The A02-A5 asset accounts represent tangible objects with both monetary and energy profiles and A06-A08 are financial contracts (liabilities) with external parties that will be central in the attribution of energy footprints. From an energy perspective A01 is immaterial, A02-A05 have intrinsic embodied energy and A06-A08 have both embodied and physical attributed energy. In particular A02 (the solar panel installation) can produce physical energy, while A03 (the battery) can store physical energy. In more realistic accounting applications the management inventory of economic artifacts will be much more detailed. In addition, the financial side will be much more granular.

Boldface accounts indicate liability accounts (they hold positive values but are summed with a negative sign). The chart of accounts can be summarized using a list of symbols as:

$$V = \begin{bmatrix} C & F & S & M & I & \mathbf{P} & \mathbf{L} & \mathbf{K} \end{bmatrix} \tag{4}$$

In summary, in the EWC integrated energy accounting system there are $N = 8$ number of accounts, the number of measured qualities ranges in $q \in (1, 2, 3)$. The accounts must balance for all three measured qualities (monetary, embodied energy and physical energy).

3.3 The evolution of the EcoWidgetCo accounting state

Now we get down to real business. The meaning and function of these accounts and values will become clear as the entity (EWC) starts transacting and updating it accounts accordingly. We will examine a total of ten transactions that are assumed to be happening in sequence (the precise timing is irrelevant for the toy model we consider here). The list of transactions is indicated here:

1. Initial Equity Transaction
2. Acquire Facilities
3. Acquire Raw Materials on Credit
4. Self-Produce Solar Energy
5. Procure Grid Electrical Energy

6. Produce Widgets (Material Processes)
7. Produce Widgets (Energy Processes)
8. Widget Sale
9. Debt Repayment
10. New Bank Loan
11. . . . Repeat . . .

Let us now examine each one in sequence.

3.3.1 Transaction 1: Initial Equity Transaction

At the initial time ($t = 0$) EcoWidgetCo is incorporated as a for-profit company by a number of enterprising individuals who are the initial shareholders. Their contribution of initial capital c_0 populates the EcoWidgetCo's bank account (becomes available cash).

Strictly speaking the bank account is an *external account*, thus it is a potentially risky asset that is linked to the health of the private bank providing the account. By providing the deposit account and receiving in exchange, e.g. fees the bank is already entangled (enabling) the business model and environmental footprint of EWC. In fully developed energy accounting and attribution this aspect should be addressed consistently. Yet here we will avoid this complication. Theoretically cash could be *paper notes* e.g., issued by a government treasury. This small change alters the shape of the economic network and creates a different pattern of attribution (it is now the *State* that enables the EWC business model). For simplicity we will treat cash as an internal asset that is decoupled from the external financial system.

With the injection of equity capital into the company we have the first business transaction! This capital injection (contribution) from external entities (shareholders) is amending the initial "null" balance sheet, the blank slate of the initial state of accounts. The transaction matrix T_1 representing this event is:

$$T_1 = \begin{bmatrix} \Delta C & \Delta F & \Delta S & \Delta M & \Delta I & \Delta \mathbf{P} & \Delta \mathbf{L} & \Delta \mathbf{K} \\ c_0 & 0 & 0 & 0 & 0 & 0 & 0 & c_0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{5}$$

A note on notation here: In general many accounting states and transactions will have significant numbers of zero elements (indicating that the account is not populated or that there is no change). For brevity we will be only displaying the *non-zero* transaction elements. It is understood that all non-displayed accounts have zero values. With this convention in our example we can more economically present the first transaction T_1 as:

$$T_1 = \begin{bmatrix} \Delta C & \Delta \mathbf{K} \\ c_0 & c_0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{6}$$

The transaction has two transaction legs. The first hits the cash account (augmenting it by the amount c_0, while the second leg hits the equity account, augmenting it by exactly the same amount). This transaction represents pure contractual activity. It establishes the fundamental role of shareholders as the residual claimants of a firm's assets. The transaction does not involve any directly measured amounts of physical of embodied energy. In principle contractual activity will change the attribution of physical and embodied energy to liability accounts but due to the nascent stage of the balance sheet (there are no owned physical artifacts inside the company's economic perimeter) these effect are still null.

The transaction T_1 updates the initial (null) state V_0 of the accounts by populating the cash and equity accounts and producing the state V_1 as the simple matrix sum:

$$V_1 = V_0 + T_1 \tag{7}$$

In terms of scalar values this is spelled out explicitly as:

$$c_0 = 0 + c_0 \tag{8}$$
$$k_0 = 0 + c_0 \tag{9}$$

As a result, the state of accounts at $t = 1$ is described by the matrix:

$$V_1 = \begin{bmatrix} C & F & S & M & I & \mathbf{P} & \mathbf{L} & \mathbf{K} \\ c_0 & 0 & 0 & 0 & 0 & 0 & 0 & k_0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{10}$$

It may be useful to highlight the nature of the "measurement" that has taken place in this transaction. The monetary values of the cash and capital accounts at this stage are based on the face value of the cash injected into the company (e.g. the numbers on bills or the digital value of a private bank account). Since equity is the only liability the cash asset value is wholly attributed to the equity account. These two monetary accounts obviously balance by construction.

In the general case the measurement matrix V capturing the values of all accounts satisfies the *balance equation for all measured qualities* q. Using explicit variables the balance conditions read:

$$C + F + S + M + I - \mathbf{P} - \mathbf{L} - \mathbf{K} = \mathbf{0} \tag{11}$$

In our EWC example and with the current state of accounts the balance sheet or statement of position reads simply $c_0 - k_0 = 0$.

On the energy measurement side, at this moment physical energy and embodied energy are only *future potentialities* that cannot be meaningfully quantified. Business scenarios with material energy profiles are implied or anticipated by the business model and reflected in the structure of the chart of accounts but are not attributable in any meaningful way.

For concreteness, using $c_0 = 50\,\mathrm{CU}$ we obtain the accounts matrix:

$$V_1 = \begin{bmatrix} C & F & S & M & I & \mathbf{P} & \mathbf{L} & \mathbf{K} \\ 50 & 0 & 0 & 0 & 0 & 0 & 0 & 50 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{12}$$

3.3.2 Transaction 2: Acquire Production Facilities

In the next transaction we imagine EWC acquiring the fabled *Means of Production*. We imagine the entity gearing up for production of its innovative micro-mobility widgets as per the business plan. This requires to acquire in the market a number of tangible (physical) assets using its newly available cash. These required production assets will be acquired from external economic actors. The important aspect from the point of view of integrated energy accounting is that they will also bring embodied energy into the entity's organizational perimeter.

In this second transaction EWC acquires i) a set of solar panels and associated battery storage and ii) a widget production facility (a widget factory). The entire investment has monetary worth v_1 and embodied energy measured at e_1. The monetary value worth of this equipment is established through the market transactions (the purchase price). We might imagine that the embodied energy of the solar panels and other equipment is provided by accompanying certificates.

The transaction type economically falls under Trading Activity and it involves material exchange of embodied energy, though no physical energy. To keep track of the embodied energy aspects, the transaction has three transaction legs (one for each account affected) whereas under conventional accounting it would have only two legs. Concretely, we have transaction T_2 represented as:

$$T_2 = \begin{bmatrix} \Delta C & \Delta F & \Delta \mathbf{K} \\ -v_1 & v_1 & 0 \\ 0 & 0 & 0 \\ 0 & e_1 & e_1 \end{bmatrix} \tag{13}$$

Let us unpack what is happening here.

1. The purchase of the equipment is done with available cash. This reduces the cash account C by v_1. Notice that for brevity we do not check if the cash account has sufficient funds for this acquisition spree.

2. The factory account F is augmented by the amount spent to acquired the factory.

3. From the perspective of the entity the transaction has zero monetary Profit or Loss. Hence there is no change in the monetary side of the Equity account $\mathbf{K}$ or ($\Delta \mathbf{K} = \mathbf{0}$).

4. The face value v_1 of cash spent cancels precisely the value of equipment. This illustrates the basic double entry identity.

5. The embodied energy of the factory account F is *attributed* the combined embodied energy of solar panels and machinery. Embodied energy is simply additive.

6. The *embodied energy change* on the asset side (e_1) is positive and is attributed wholly to equity. This simple rule is possible because equity is the only liability present. We'll see below that things may get more complicated in the presence of more liabilities.

All layers of the second transaction balance as required. The physical energy layer balances trivially, being still null. Posting the transaction to the ledger V results in the second change of state:

$$V_2 = \begin{bmatrix} C & F & S & M & I & \mathbf{P} & \mathbf{L} & \mathbf{K} \\ c_0 & v_1 & 0 & 0 & 0 & 0 & 0 & k_0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & e_1 & 0 & 0 & 0 & 0 & 0 & e_1 \end{bmatrix} \tag{14}$$

Numerically, with $v_1 = 10\,\mathrm{CU}$ and $e_1 = 100\,\mathrm{EU}$ the EWC accounting state becomes:

$$V_2 = \begin{bmatrix} C & F & S & M & I & \mathbf{P} & \mathbf{L} & \mathbf{K} \\ 40 & 10 & 0 & 0 & 0 & 0 & 0 & 50 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 & 0 & 0 & 0 & 100 \end{bmatrix} \tag{15}$$

3.3.3 Transaction 3: Acquire Raw Materials on Credit

The third transaction will be a further asset acquisition story, but with a twist that helps us explore a new complexity: EWC decides to acquire consumable raw materials (worth v_2 and having embodied energy e_2). This purchase is not made using cash but on (trade) credit. Let us imagine that the reason for that is that the entity's management decides it is good to establish a credit profile with the market. Alternatively may it wants to preserve cash for managing liquidity under uncertain circumstances.

Whatever the motivation, the transaction will create a new monetary liability with an external party. The trade credit obtained in this way must be repaid in the future (likely with added interest but we will ignore this here). This transaction also falls under Trading Activity and the raw material will have embodied energy qualities but no physical energy.

Importantly for our purposes, the transaction being on credit and creating a new liability will raise the question of how the embodied energy newly entering the organizational perimeter will be attributed, in light of the pre-existing assets and liabilities of the company.

The three transaction legs are as follows:

$$T_3 = \begin{bmatrix} \Delta M & \Delta \mathbf{P} & \Delta \mathbf{K} \\ v_2 & v_2 & 0 \\ 0 & 0 & 0 \\ e_2 & f_1 e_2 & (1 - f_1) e_2 \end{bmatrix} \tag{16}$$

In the above representation we explicitly indicate a *parameterized attribution* of the embodied energy e_2 to the existing liabilities (accounts payable $\mathbf{P}$ and equity $\mathbf{K}$) using the attribution factor f_1. This helps illustrate the methodological freedom that exists in attribution schemes. This freedom is not even limited to a linear approach that we explore here. In a concrete and detailed integrated accounting context this flexibility must be reduced and channeled, e.g., through specific accounting standards and prescriptions that indicate the available range of options in each attribution instance.

Plausible options for embodied energy attribution in this case might be:

1. $f_1 = 1$, attribute the newly acquired embodied energy of the raw material entirely to the external entity that provided the credit

2. $f_1 = 0$, attribute the embodied energy entirely to shareholders

3. $f_1 = v_2/(c_1 + v_1)$, attribute the embodied energy pro-rata the monetary size of the new liability versus the currently existing liabilities

Any of the options must satisfy the embodied energy balance equation that ensures that newly acquired embodied energy entering on the asset side is reflected exactly also on the liability side. From an implementation perspective an important aspect of the third option (the pro-rata attribution to existing liabilities) is that for the numerical update of the account states (the posting the transaction) one requires *retrieving* the current state of accounts and using their monetary values to derive suitable ratios before one can compute the actual numerical values of the transaction. While traditional accounting transactions are generally self-contained operations (all the numerical data is fixed and contained in the transaction), depending on methodology, the attribution of energy to liabilities may require access to all the accounting system data. This introduces business logic that is similar, e.g. to valuation methodologies, hence relying on procedures that are not fully implemented within the database system.

The result of posting the transaction under the third option (pro-rata attribution) on the EWC accounts is symbolically:

$$V_3 = \begin{bmatrix} C & F & S & M & I & \mathbf{P} & \mathbf{L} & \mathbf{K} \\ c_1 & v_1 & 0 & v_2 & 0 & v_2 & 0 & k_0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & e_1 & 0 & e_2 & 0 & f_1 e_2 & 0 & e_1 + (1 - f_1) e_2 \end{bmatrix} \tag{17}$$

or numerically for ($v_2 = 15$, $e_2 = 150$):

$$V_3 = \begin{bmatrix} C & F & S & M & I & \mathbf{P} & \mathbf{L} & \mathbf{K} \\ 40 & 10 & 0 & 15 & 0 & 15 & 0 & 50 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 100 & 0 & 150 & 0 & 45 & 0 & 205 \end{bmatrix} \tag{18}$$

Now that we have a sequence of transactions when can take stock of what has happened over the entire period:

$$V_3 = \begin{bmatrix} C & F & S & M & I & \mathbf{P} & \mathbf{L} & \mathbf{K} \\ c_1 & v_1 & 0 & v_2 & 0 & v_2 & 0 & k_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & e_1 & 0 & e_2 & 0 & eb_1 & 0 & eb_2 \end{bmatrix} \tag{19}$$

where

$$c_1 = c_0 - v_1 \tag{20}$$

$$k_1 = k_0 \tag{21}$$

$$eb_1 = f_1 e_2 \tag{22}$$

$$eb_2 = e_1 + (1 - f_1) e_2 \tag{23}$$

1. The fist row of V_3 is essentially the monetary value balance sheet at the end of the period. In a different more detailed format this would lead to the statement of financial position.
2. The second row of V_3 is the primary energy energy balance sheet (currently zero). Notice that this is because while embodied energy has changed hands in the three transactions of this period, there has been no physical energy flow in or out of the entity perimeter (this is of course a simplification focusing on materiality and clarity. For sure the EWC headquarters must have received an electricity bill by now!).
3. The third row of V_3 is the embodied energy balance sheet which attributes the embodied energy of equipment and materials to the current liabilities (the equity and accounts payable accounts)
4. The transactions and their impact on accounts can also be grouped and reported as Profit and Loss or Income Statements. There are only explanatory.

3.3.4 Transaction 4: Self-Produce Solar Energy

EWC aims to produce mobility widgets and to that end it needs to source primary energy. EWC will use the next period to power up. We imagine the entity charging the acquired battery installation (Transaction 2) by operating its own solar panel facility and storing the produced electricity in its battery installation and, in addition, acquiring electricity from external providers (to keep the widget production going during these overcast days).

For accounting purposes the purchased electricity is also assumed stored (becomes an measured quality) of the battery facility. In reality the temporal utilization patterns of economic entities are activity specific, e.g., both self-generated and grid power might be used on ongoing basis rather than stored. Just like regular accounting, integrated energy accounting systems are based on stock variables and thus there is a need to integrate continuous activities over time. In reality the actual activity might involve variable flows in and out of the entity perimeter.

The production of electricity from solar power encodes the operation of solar panels. For a touch of realism we might assume an associated monetary amount of nominal fixed expenses x required for operating the solar panel installation over a certain period. The ensuing charging of the batteries will provide EWC with physical energy worth v_3. The energy content is denoted $\mathcal{E}_1$.

$$T_4 = \begin{bmatrix} \Delta C & \Delta S & \Delta \mathbf{K} \\ -x & v_3 & v_3 - x \\ 0 & \mathcal{E}_1 & \mathcal{E}_1 \\ 0 & 0 & 0 \end{bmatrix} \tag{24}$$

In the above, the physical energy amount E_1 that is generated by the solar panels is added to the physical energy leg of the energy storage account S. This reflects that physical energy enters the economic perimeter of the entity and becomes a measured quality of its assets. What about the physical energy balance and liability attribution? The counterparty to the energy generation transaction via solar panels is not another economic agent. Similar to wealth generated by agricultural activity, the source in this instance is the Earth's environment / ecosystem. A simple approach is to attribute the physical energy to the shareholder account. A more sophisticated approach could be more informative in an economic network context and it would require to introduce an environmental account that would serve as a source of physical energy and eventual repository of embodied energy.

Incidentally, we have here another example of different valuation options: The value of the solar energy obtained by operating the solar panels (v_3) as obtained from current market prices of energy is assumed to exceed the cost of operation x, again measured at current market rates for the expenses involved. We could thus recognize the difference as profit immediately, by "marking-to-market" the procured energy. Alternatively we could measure it at cost (hence x) and wait to recognize the monetary gain only once (and if) any widgets manufactured using this energy have been sold. In the above choice we effectively immediately book this "unrealized gain" as income $v_3 - x$.

The accounting state of the entity now becomes:

$$V_4 = \begin{bmatrix} C & F & S & M & I & \mathbf{P} & \mathbf{L} & \mathbf{K} \\ c_1 - x & v_1 & v_3 & v_2 & 0 & v_2 & 0 & k_1 + v_3 - x \\ 0 & 0 & \mathcal{E}_1 & 0 & 0 & 0 & 0 & \mathcal{E}_1 \\ 0 & e_1 & 0 & e_2 & 0 & eb_1 & 0 & eb_2 \end{bmatrix} \tag{25}$$

or numerically for ($x = 5$, $v_3 = 20$, $\mathcal{E}_1 = 30$):

$$V_4 = \begin{bmatrix} C & F & S & M & I & \mathbf{P} & \mathbf{L} & \mathbf{K} \\ 35 & 10 & 20 & 15 & 0 & 15 & 0 & 65 \\ 0 & 0 & 30 & 0 & 0 & 0 & 0 & 30 \\ 0 & 100 & 0 & 150 & 0 & 45 & 0 & 205 \end{bmatrix} \tag{26}$$

3.3.5 Transaction 5: Procure Grid Electrical Energy

As next action during this period, EWC acquires in the market against cash additional physical energy $\mathcal{E}_2$ worth v_4. That value is now assigned internally to an "energy storage" account S and in practice it is ready to be used in production.

$$T_5 = \begin{bmatrix} \Delta C & \Delta S & \Delta \mathbf{K} \\ -v_4 & v_4 & 0 \\ 0 & \mathcal{E}_2 & \mathcal{E}_2 \\ 0 & 0 & 0 \end{bmatrix} \tag{27}$$

In this instance physical energy is acquired from the economic network. The cash transaction with an external energy provider means there is no generation of new liabilities. What should be the matching liability account that balances the physical energy inflow? One thought might be to introduce a new account, representing the energy provider. While the cash payment is obviously to that external party, the monetary side of accounts is already balanced internally (between the internal accounts of cash and energy stock). Recording the external value transfer to the energy provider would be valid in the context of quadruple-entry accounting. The full books would balance with a compensating reduction of value from the energy provider's energy stock account. A simpler (but less informative approach) is to attribute the energy inflow to equity. The resulting accounting state will now be:

$$V_5 = \begin{bmatrix} C & F & S & M & I & \mathbf{P} & \mathbf{L} & \mathbf{K} \\ c_1 - x - v_4 & v_1 & v_3 + v_4 & v_2 & 0 & v_2 & 0 & k_1 + v_3 - x \\ 0 & 0 & \mathcal{E}_1 + \mathcal{E}_2 & 0 & 0 & 0 & 0 & \mathcal{E}_1 + \mathcal{E}_2 \\ 0 & e_1 & 0 & e_2 & 0 & eb_1 & 0 & eb_2 \end{bmatrix} \tag{28}$$

or, numerically, assuming $v_4 = 12$, $\mathcal{E}_2 = 60$,

$$V_5 = \begin{bmatrix} C & F & S & M & I & \mathbf{P} & \mathbf{L} & \mathbf{K} \\ 23 & 10 & 32 & 15 & 0 & 15 & 0 & 65 \\ 0 & 0 & 90 & 0 & 0 & 0 & 0 & 90 \\ 0 & 100 & 0 & 150 & 0 & 45 & 0 & 205 \end{bmatrix} \tag{29}$$

At the end of the period we might again summarize the transactions:

$$V_5 = \begin{bmatrix} C & F & S & M & I & \mathbf{P} & \mathbf{L} & \mathbf{K} \\ c_2 & v_1 & v_5 & v_2 & 0 & v_2 & 0 & k_2 \\ 0 & 0 & \mathcal{E}_3 & 0 & 0 & 0 & 0 & \mathcal{E}_3 \\ 0 & e_1 & 0 & e_2 & 0 & eb_1 & 0 & eb_2 \end{bmatrix} \tag{30}$$

where

$$c_2 = c_1 - x - v_4 \tag{31}$$
$$\mathcal{E}_3 = \mathcal{E}_1 + \mathcal{E}_2 \tag{32}$$
$$k_2 = k_1 + v_4 - x \tag{33}$$
$$v_5 = v_3 + v_4 \tag{34}$$

The fist row of V_5 reflects the new value balance. The second and third row of V_5 is the energy balance. On the asset side it includes both embodied energy and physical energy.

3.3.6 Transaction 6: Produce Widgets (Material Processes)

The objective of all the capital investment is of-course to produce and sell widgets at profit. So the next few transactions aim to capture the process of production and trade of goods and services. Capturing the gist of what goes on from both a value and energy perspective requires at least two transactions per production process. The first transaction captures the internal monetary adjustments as resources are shifted and re-combined. The second transaction captures the internal generation of embodied energy. Combining these two steps in a single compound transaction with multiple legs is possible but is less transparent.

The sixth transaction will thus encode the production process of widgets. Of-course, given the innumerable details of production processes such accounting will be highly stylized. Yet it is important to establish some key mechanisms at play here: Production of goods and services happens through the consumption of raw materials (and also, obviously, labor, but this dimension is left out presently). In the process physical energy is converted to (useful) work and (generally) wasted heat (though energy recovery is becoming increasingly common).

The transaction is entirely internal (in particular it does not affect any liabilities or equity accounts). It has three legs, capturing changes to the energy stock, the material stock and the inventory account. Monetary, physical and embodied energy values of the ingredients get merged into a final product. This is captured in transaction T_6:

$$T_6 = \begin{bmatrix} \Delta S & \Delta M & \Delta I \\ -v_5 & -v_2 & v_5 + v_2 \\ -\mathcal{E}_3 & 0 & \mathcal{E}_3 \\ 0 & -e_2 & e_2 \end{bmatrix} \tag{35}$$

which produces the new accounting state:

$$V_6 = \begin{bmatrix} C & F & S & M & I & \mathbf{P} & \mathbf{L} & \mathbf{K} \\ c_2 & v_1 & 0 & 0 & v_5 + v_2 & v_2 & 0 & k_2 \\ 0 & 0 & 0 & 0 & \mathcal{E}_3 & 0 & 0 & \mathcal{E}_3 \\ 0 & e_1 & 0 & 0 & e_2 & eb_1 & 0 & eb_2 \end{bmatrix} \tag{36}$$

which numerically, (with $v5 = 32$, $v_2 = 15$, $E_3 = 90$, $e_2 = 150$) produces:

$$V_6 = \begin{bmatrix} C & F & S & M & I & \mathbf{P} & \mathbf{L} & \mathbf{K} \\ 23 & 10 & 0 & 0 & 47 & 15 & 0 & 65 \\ 0 & 0 & 0 & 0 & 90 & 0 & 0 & 90 \\ 0 & 100 & 0 & 0 & 150 & 45 & 0 & 205 \end{bmatrix} \tag{37}$$

Asset and liabilities maintain their balance across all measured qualities.

3.3.7 Transaction 7: Produce Widgets (Energy Processes)

The second part of production process deals with the flow of energy within the entity and has no affinity with typical monetary operations. Production is a physical transformation that,among others, converts (uses up) physical energy to embodied energy. Accounting correctly for this process is thus essential for keeping track of embodied energy in general.

In principle some of the physical energy is utilized (becomes useful energy producing work) while the rest is fully or partially wasted energy that dissipates into the environment. The distinction between useful and waste energy is not essential for the most basic accounting of embodied energy in goods or services. It does becomes relevant though in more refined discussions that focus on energy efficiency and energy intensity.

While the transactions we examined so far shift like-for-like values "horizontally" between accounts, this transaction is special in that it shifts energy values "vertically" within an account. This type of transaction is obviously not present in a pure monetary accounting system where the only attribute of economic artifacts is their value. The transaction is an example of a *stoichiometric equation* that establishes a relationship between the quantities of reactants and products. In this case it simply a one-to-one conversion of all physical energy to embodied energy. The transaction has two legs, one affecting the inventory account and one adjusting the liability side.

$$T_7 = \begin{bmatrix} \Delta I & \Delta \mathbf{K} \\ 0 & 0 \\ -\mathcal{E}_3 & -\mathcal{E}_3 \\ \mathcal{E}_3 & \mathcal{E}_3 \end{bmatrix} \tag{38}$$

Let us examine the asset side first, which in this simple transaction involves only the widget inventory account. This transaction has no monetary footprint. The monetary aspect of production was captured in the previous transaction. The essence of the transaction is very simple: an amount of physical energy is converted into embodied energy.. Yet this simple calculus captures two important thermodynamical laws:

- The total energy (sum of all types of physical and embodied energy) on the asset side must remain constant. This is the first law of thermodynamics which applies to purely internal transactions (that do not involve inflow of physical or embodied energy from external actors or the environment).
- The total embodied energy of the asset side can only increase (again for purely internal transactions). This is effectively the second law of thermodynamics. This constrain embeds into integrated accounting a thermodynamic arrow of time. The opposite transaction (embodied energy converted to primary energy) is not allowed. By inspection of an ordered set of business transactions we can tell in which physical time order they have occurred. Regular accounting transactions are time symmetric - there is nothing that prohibits the mirror (reverse) of any given transaction.[1]

The liability side (external accounts) are not actively involved in this transaction. Yet the shift from physical to embodied energy must be reflected (so that physical and embodied energy continue balancing across assets and liabilities). In the presence of a variety of liabilities that rebalancing of energy accounts must also be properly attributed to liabilities.

Symbolically the state of accounts is now:

$$V_7 = \begin{bmatrix} C & F & S & M & I & \mathbf{P} & \mathbf{L} & \mathbf{K} \\ c_2 & v_1 & 0 & 0 & v_5 + v_2 = v_6 & v_2 & 0 & k_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & e_1 & 0 & 0 & e_2 + \mathcal{E}_3 = e_3 & eb_1 & 0 & eb_2 + \mathcal{E}_3 \end{bmatrix} \tag{39}$$

which numerically with $\mathcal{E}_3 = 90$ produces:

$$V_7 = \begin{bmatrix} C & F & S & M & I & \mathbf{P} & \mathbf{L} & \mathbf{K} \\ 23 & 10 & 0 & 0 & 47 & 15 & 0 & 65 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 & 240 & 45 & 0 & 295 \end{bmatrix} \tag{40}$$

[1] There are manifestations of a defined temporal arrow in, e.g., the depreciation of assets reflecting entropy increase, but this information and direction is provided externally to the accounting system. One could equally well introduce appreciating asset values.

3.3.8 Transaction 8: Widget Sale

We have finally made it to the point that EWC will fulfill its business model: it will sell its product into a (hopefully) eager market for mobility widgets. The transaction will book cash receipts to the cash account and empty the widget inventory. The new element that comes with integrated energy accounting is that the sale *transfers* the widget's embodied energy to others.

Three accounts are affected here: the cash account, the inventory account and equity:

$$T_8 = \begin{bmatrix} \Delta C & \Delta I & \Delta \mathbf{K} \\ v_7 & -v_6 & v_7 - v_6 \\ 0 & 0 & 0 \\ 0 & -e_3 & -e_3 \end{bmatrix} \tag{41}$$

We assume that EWC sells its widgets to an eager market or more precisely that the monetary cash inflow exceeds past monetary outflows ($v_7 > v_6$). Hence the monetary surplus is booked as a profit (which becomes an addition to equity).

$$V_8 = \begin{bmatrix} C & F & S & M & I & \mathbf{P} & \mathbf{L} & \mathbf{K} \\ c_2 + v_7 & v_1 & 0 & 0 & 0 & v_2 & 0 & k_2 + v_7 - v_6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & e_1 & 0 & 0 & 0 & eb_1 & 0 & eb_2 + \mathcal{E}_3 - e_3 \end{bmatrix} \tag{42}$$

which produces:

$$V_8 = \begin{bmatrix} C & F & S & M & I & \mathbf{P} & \mathbf{L} & \mathbf{K} \\ c_3 & v_1 & 0 & 0 & 0 & v_2 & 0 & k_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & e_1 & 0 & 0 & 0 & eb_1 & 0 & eb_3 \end{bmatrix} \tag{43}$$

where

$$c_3 = c_2 + v_7 \tag{44}$$
$$eb_1 = f_1 e_2 \tag{45}$$
$$eb_3 = eb_2 - e_2 = e_1 - f_1 e_2 \tag{46}$$
$$k_3 = k_2 + v_7 - v_6 \tag{47}$$

or numerically, with $v_7 = 60$, $v_6 = 47$ and $e_3 = 240$

$$V_8 = \begin{bmatrix} C & F & S & M & I & \mathbf{P} & \mathbf{L} & \mathbf{K} \\ 83 & 10 & 0 & 0 & 0 & 15 & 0 & 78 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 & 0 & 45 & 0 & 55 \end{bmatrix} \tag{48}$$

Notice that after the widgets are produced and sold, the embodied energy inside the entity (and thus also what is attributed to its liabilities) is reduced back to the baseline profile (the embodied energy of its fixed assets). In the broader economic network context the embodied energy of the widgets has been transferred to the widget buyers (the final demand for the widgets).

3.3.9 Transaction 9: Debt Repayment

In the ninth transaction EWC fulfills a contractual obligation and repays its trade credit that was established in the third transaction. Given our scenario of events there are, indeed, sufficient funds to do this (no credit event). The accounts payable liability facilitated (had responsibility for) the widget production and was attributed a specific fraction of embodied energy of that raw material. This treatment of Accounts Payable is a blueprint for more complex contractual arrangements such as procurement contracts where the entity agrees to deliver or acquire goods or services with defined embodied energy over multi-period time intervals.

Repaying the funds extinguishes the monetary obligation to the external party. This also removes the attribution of energy in its entirety. Now that the liability has been extinguished the attribution of the energy footpring to the remaining liabilities must be re-balanced. In terms of the embodied energy transaction this requires shifting embodied energy amounts to the remaining liabilities. Given the only remaining liability is equity, it will get attributed all embodied energy that was previously attributed to the accounts payable.

$$T_9 = \begin{bmatrix} \Delta C & \Delta \mathbf{P} & \Delta \mathbf{K} \\ -v_2 & -v_2 & 0 \\ 0 & 0 & 0 \\ 0 & -eb_1 & eb_1 \end{bmatrix} \tag{49}$$

The result is symbolically:

$$V_9 = \begin{bmatrix} C & F & S & M & I & \mathbf{P} & \mathbf{L} & \mathbf{K} \\ c_3 - v_2 & v_1 & 0 & 0 & 0 & 0 & 0 & k_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & e_1 & 0 & 0 & 0 & 0 & 0 & eb_3 - eb_1 \end{bmatrix} \tag{50}$$

or numerically ($v_2 = 15$, $eb_1 = 45$)

$$V_9 = \begin{bmatrix} C & F & S & M & I & \mathbf{P} & \mathbf{L} & \mathbf{K} \\ 68 & 10 & 0 & 0 & 0 & 0 & 0 & 78 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 & 0 & 0 & 0 & 100 \end{bmatrix} \tag{51}$$

3.3.10 Transaction 10: New Bank Loan

In the next and final transaction, we imagine that EWC management feels that its business model is now validated. It wants to accelerate its production cycle by raising debt capital. In the tenth transaction we examine what happens if the entity borrows funds from a financial intermediary (e.g. a bank). Specifically, EWC, borrows funds worth v_8, creating a long-term loan liability. The transaction looks as follows:

$$T_{10} = \begin{bmatrix} \Delta C & \Delta \mathbf{L} & \Delta \mathbf{K} \\ v_8 & v_8 & 0 \\ 0 & -f_3 eb_4 & (-1 + f_3) eb_4 \end{bmatrix} \tag{52}$$

In the above the attribution of embodied energy to liabilities is again done using a linear attribution function using the factor f_3. The borrowing does not bring new embodied energy into the entity that can be readily associated with it. Yet, as with the initial equity injection transaction, it is an event pregnant with *future* energy liabilities. The borrowing will be an enabler of future energy consumption and GHG emissions .

The monetary accounting in this transaction is standard: the cash account is credited, and the loan account (an external account) captures the new liability. It is again the attribution of energy to liabilities that is subject to various options. The loan is assumed a general purpose loan hence the plausible options in these circumstances are:

- $f_3 = 0$ absolve the new liability from any energy attribution (in favor of attributing to equity)
- $f_3 = v_8/v_T$ attribute the existing energy footprint pro-rata the size of the loan versus the total asset base. This has the effect of diluting the existing attribution to equity: the different liability holders now effectively share the footprint.

This brings us to the final state:

$$V_{10} = \begin{bmatrix} C & F & S & M & I & \mathbf{P} & \mathbf{L} & \mathbf{K} \\ c_4 & v_1 & 0 & 0 & 0 & 0 & v_8 & k_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & e_1 & 0 & 0 & 0 & 0 & eb_5 & eb_6 \end{bmatrix} \tag{53}$$

where

$$c_4 = c_3 + v_8 - v_2 \tag{54}$$
$$k_4 = k_3 \tag{55}$$

or numerically, with $v_8 = 32$, $eb_4 = 100$

$$V_{10} = \begin{bmatrix} C & F & S & M & I & \mathbf{P} & \mathbf{L} & \mathbf{K} \\ 100 & 10 & 0 & 0 & 0 & 0 & 31 & 78 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 100 & 0 & 0 & 0 & 0 & 41 & 59 \end{bmatrix} \tag{56}$$

4 energyLedger: Relational Database Implementation of Integrate Energy Accounting Principles

In this section we describe how the framework of integrated energy accounting introduced in [4] and further discussed and illustrated in the previous sections can be implemented using the concepts and tools of relational databases. The conceptual super-set relation between conventional and integrated accounting already carries over also to practical implementations: An integrated energy accounting database or system is, in principle, a *generalization* of a conventional system: It could conceivably be used for financial accounting without ever populating the energy dimensions.

4.1 Implementation in Postgres

Digital implementations of integrated accounting (or conventional accounting for that matter!) are in principle highly diverse as the conceptual requirements are rather high level.

- There is flexibility in terms of implementing features and behaviors either at the *application layer* (also named programming or business logic), versus implementing such features directly *within the database* system itself.
- Using specifically a *relational database* from the much broader class of database systems is not a pre-condition for implementing integrated accounting. Other systems that offer persistent storage and (depending on requirements) various guarantees on data consistency and integrity are also viable options.
- While there is a certain degree of standardization around relational databases (e.g. the SQL:2016 or ISO/IEC 9075:2016 standard) yet database features that are important in practical implementations may vary between different RDBMS. This may affect for example the manner in which accounting logic (rules, triggers) are implemented using such internal scripting languages.
- There are different possible technical approaches to constructing database schemas. Differences in schema design become relevant when scale, performance and flexibility requirements start competing with each other.

Despite this range of options, a concrete realization will help us illustrate how integrated energy accounting can be implemented in the context of currently available open source platforms. We will use a specific relational database, PostrgreSQL to illustrate the implementation of integrated energy accounting. PostgreSQL is a long-lived, open source, object-relational database system that uses and extends the SQL language. The origins of PostgreSQL date back to 1986 as part of the POSTGRES project at the University of California at Berkeley. PostgreSQL is appreciated widely for its architecture, reliability, data integrity, robust feature-set and extensibility. It runs on all major operating systems, is ACID-compliant, and has add-ons such as the popular PostGIS geospatial database extender.

In the next sections we will go step-by-step over the energyLedger implementation in Postgres. A preview of the key steps might be useful:

- The concept of (ledger) accounts realized as a database table.
- The concept of transactions realized as a database table.
- The concept of transaction legs or records) also realized as a database table.
- The concept of a double-entry balance preserving database triggers.
- The concept of conservation law preserving database triggers.

Note that in any production oriented system one would have to introduce significant numbers of additional structures and logic. For the present proof-of-principle these details would only obscure the main extensions required for implementing integrated energy accounting.

4.2 The Database Schema

4.2.1 The Accounts Table

The **Account** table helps *categorize* the measured values captured by the ledger system. Accounts are generally modeled as a hierarchical tree or taxonomy. The top most levels point to major balance sheet accounts categories and second or lower levels provide an arbitrarily refined taxonomy. The Account table represents the core index of measurable items in the accounting system. This table does not hold numerical data. It only catalogs relevant attributes of accounts. Account attributes will be names, codes, types of various forms (e.g Internal versus External Account Type) etc.

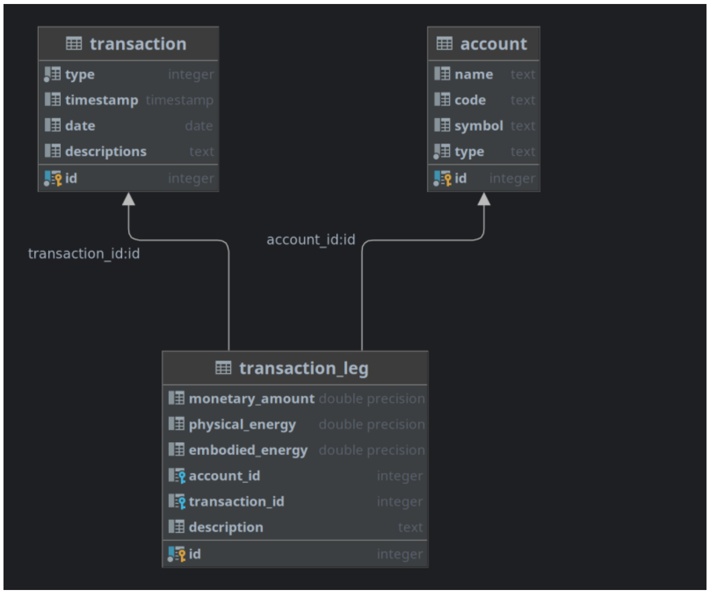

Figure 2: The simplified and stylized schema consists of only three tables: accounts, transactions and transaction legs. Besides a number of required attributes per table, an important feature are the foreign key relations from the Transaction Leg table to the Accounts and Transaction tables. These features are not specifically due to integrated energy accounting. The features that *are* unique are multiple measured qualities in transaction legs (monetary, physical energy, embodied energy) and a requirement to identify transactions are wholly internal to the organization or external.

```
Generate Table Account

create table account
(
account_id serial primary key,
name   text,
code   text,
account_number integer,
symbol text,
type   text
);
```

The above SQL statement creates the Account table but we need to populate it with our chart of accounts. This might be a sequence of insert statements as per below (simplified for legibility). This will only be done when initializing the accounting system.

```
Populate Chart of Accounts

INSERT INTO account (account_id, name) VALUES (1, 'Cash');
INSERT INTO account (account_id, name) VALUES (2, 'Property');
INSERT INTO account (account_id, name) VALUES (3, 'Loan');
...
```

Notice that at this stage there is no constraint as to what type of accounts populate the Chart of Accounts, what is their relation etc. A Chart of Accounts is a very flexible construct and can be adapted in various ways to make integrate energy accounting more informative.

4.2.2 The Transactions Table

Transactions are at the core of an accounting system. A transaction is a single record in the accounting ledger. It is the conceptual representation of a self-contained business transaction. Other names used to express the concept are *Journal*

Entry. Conceptually transactions are *sparse matrices* that are applied to accounts. The sparse matrix representation is not a natural fit in the relational database design. Representing the elements of a transaction as a set of tuples (as would be the natural design) would be cumbersome and wasteful.

The **Transaction** table does not hold the actual numerical transaction data. It works, effectively, as a reference (key) for transaction data sets that will be stored separately as the "legs" or components of the transaction. This design allows for an arbitrary number of transaction legs per transaction and overall easier accommodation of varying types of transactions. Transactions are characterized by various attributes. These include timestamps (when the business event occurred) and posting dates (when the transaction is posted to the ledger). Transactions will have type attributes (e.g. transfer, sale, purchase, repayment etc.) and different types might typically required different data sets.

A critical attribute of a Transaction record in the context of integrated energy accounting is whether the transaction is purely internal (does not involve exchanges with entities outside the organizational perimeter) or external (involves in or outflow). The internal/external attribute helps enforce additional constraints where applicable.

An example of a Transaction table creation instruction is:

```
Generate Table Transaction

create table transaction
(
id serial primary key,
timestamp timestamp,
date posting_date,
type integer,
descriptions text
);
```

Note again that a Transaction record does not indicate either amounts or, indeed which accounts are affected. That information is contained in the transaction "legs" which in turn link to both the transaction and account tables.

4.2.3 The Transaction Leg Table

The **Transaction Leg** table contains the actual transaction data (the numbers of double entry bookkeeping). Other names for these records are *Posting* or *Transaction Record*.

Each transaction leg references the Transaction it is part of, the Account it is affecting and the values to be added or subtracted from accounts along *all the measured dimensions* (monetary, different forms of energy etc.). Note that the transaction leg does not include dates. This is captured in the Transaction table.

The mechanism of adding columns to the transaction leg table means that a large number of different concurrently measured qualities of an account can be accommodated. The accounting logic is implemented through the following construct: The Transaction Leg entries associated with a Transaction entry must be successfully completed (or otherwise none can be completed). This will be the primary means to implement double entry balance constraints and thermodynamical law constraints.

```
Generate Table Transaction_Leg

create table transaction_leg
(
id serial primary key,
monetary_amount double precision,
physical_energy double precision,
embodied_energy double precision,
account_id integer constraint fk_account references account,
transaction_id  integer constraint fk_transaction references transaction,
description     text
);
```

4.3 The Balance Equation Trigger

The (generalized) balance equation trigger checks and enforces that all accounts add up to zero horizontally (across all three dimensions).

```
CREATE FUNCTION public.check_leg() RETURNS trigger
LANGUAGE plpgsql
AS $$
DECLARE
      monetary_sum DECIMAL(13,2);
      physical_sum DECIMAL(13,2);
      embodied_sum DECIMAL(13,2);
BEGIN
IF (TG_OP = 'INSERT') THEN
SELECT SUM(monetary_amount) INTO monetary_sum FROM transaction_leg
   WHERE transaction_id = NEW.transaction_id;
SELECT SUM(physical_energy) INTO physical_sum FROM transaction_leg
   WHERE transaction_id = NEW.transaction_id;
SELECT SUM(embodied_energy) INTO embodied_sum FROM transaction_leg
   WHERE transaction_id = NEW.transaction_id;
ELSE
SELECT SUM(monetary_amount) INTO monetary_sum FROM transaction_leg
   WHERE transaction_id = OLD.transaction_id;
SELECT SUM(physical_energy) INTO physical_sum FROM transaction_leg
   WHERE transaction_id = OLD.transaction_id;
SELECT SUM(embodied_energy) INTO embodied_sum FROM transaction_leg
   WHERE transaction_id = OLD.transaction_id;
END IF;
IF monetary_sum != 0 THEN
RAISE EXCEPTION
      'Sum of monetary amounts must be 0, not %', monetary_sum;
END IF;
IF physical_sum != 0 THEN
RAISE EXCEPTION
      'Sum of physical energy amounts must be 0, not %', physical_sum;
END IF;
IF embodied_sum != 0 THEN
RAISE EXCEPTION
      'Sum of embodied energy amounts must be 0, not %', embodied_sum;
END IF;
RETURN NEW;
END;
$$;
```

4.4 Energy Law Trigger

The Energy Law Trigger enforces that for internal transactions (transactions that do not see inflow or outflow of any form of energy inside the organizational perimeter) the sum of total energy across all energy species is constant.

```
CREATE FUNCTION public.check_first_law() RETURNS trigger
LANGUAGE plpgsql
AS $$
DECLARE
         energy_sum DECIMAL(13, 2);
         t_type INTEGER;
BEGIN
IF (TG_OP = 'INSERT') THEN
SELECT type INTO t_type FROM transaction WHERE NEW.transaction_id = id;
         IF (t_type = 1) THEN
         SELECT SUM(ENERGY) INTO energy_sum FROM (
         SELECT sum(transaction_leg.physical_energy) AS ENERGY
         FROM transaction_leg, account WHERE transaction_id = NEW.transaction_id
```

```
        AND account_id = account.id AND (account.type = 'AS')
        UNION
        SELECT sum(transaction_leg.embodied_energy) AS ENERGY
        FROM transaction_leg, account WHERE transaction_id = NEW.transaction_id
        AND account_id = account.id AND (account.type = 'AS')
        ) AS TMP1;
        ELSE
        energy_sum = 0;
        END IF;
ELSE
SELECT type INTO t_type FROM transaction WHERE OLD.transaction_id = id;
        IF (t_type = 1) THEN
        SELECT SUM(ENERGY) INTO energy_sum FROM (
        SELECT sum(transaction_leg.physical_energy) AS ENERGY
        FROM transaction_leg, account WHERE transaction_id = OLD.transaction_id
        AND account_id = account.id AND (account.type = 'AS')
        UNION
        SELECT sum(transaction_leg.embodied_energy) AS ENERGY
        FROM transaction_leg, account WHERE transaction_id = OLD.transaction_id
        AND account_id = account.id AND (account.type = 'AS')
        ) AS TMP2;
        ELSE
        energy_sum = 0;
        END IF;
END IF;
IF energy_sum != 0 THEN
RAISE EXCEPTION 'Energy conservation violation for internal transaction: %', energy_sum;
END IF;
RETURN NEW;
END;
$$;
```

4.5 Entropy Law Trigger

The Entropy Law Trigger enforces that for internal transactions (transactions that do not see inflow or outflow of any form of energy inside the organizational perimeter) the sum of total embodied energy across all energy species can only increase.

```
CREATE FUNCTION public.check_second_law() RETURNS trigger
LANGUAGE plpgsql
AS $$
DECLARE
        embodied_energy_sum DECIMAL(13, 2);
        t_type INTEGER;
BEGIN
IF (TG_OP = 'INSERT') THEN
        SELECT type INTO t_type FROM transaction WHERE NEW.transaction_id = id;
        IF (t_type = 1) THEN
        SELECT SUM(ENERGY) INTO embodied_energy_sum FROM (
        SELECT sum(transaction_leg.embodied_energy) AS ENERGY
        FROM transaction_leg, account WHERE transaction_id = NEW.transaction_id AND
        account_id = account.id AND (account.type = 'AS')
        ) AS TMP1;
ELSE
embodied_energy_sum = 0;
END IF;
ELSE
```

```
        SELECT type INTO t_type FROM transaction WHERE OLD.transaction_id = id;
        IF (t_type = 1) THEN
        SELECT SUM(ENERGY) INTO embodied_energy_sum FROM (
        SELECT sum(transaction_leg.embodied_energy) AS ENERGY
        FROM transaction_leg, account WHERE transaction_id = OLD.transaction_id AND account_id =
        ) AS TMP2;
        ELSE
        embodied_energy_sum = 0;
        END IF;
END IF;
IF embodied_energy_sum < 0 THEN
RAISE EXCEPTION 'Embodied Energy decrease for internal transaction: %', embodied_energy_sum;
END IF;
RETURN NEW;
END;
$$;
```

4.6 Transaction Data Insertion

Data insertion into an operational system will consist of inserting transactions and their associated transaction legs. A transaction can have any number of legs associated with it. The transaction legs will have numerical values for the types of measurements (money, energy) that are non-zero. The complete SQL code available at the repository has a longer list of transaction examples.

4.6.1 Example Transaction 1

```
Transaction and Transaction Legs for a Capital Injection

INSERT INTO transaction (transaction_id, description)
VALUES (1, 'Initial Equity Transaction');
INSERT INTO transaction_leg (monetary_amount, physical_energy, embodied_energy,
account_id, transaction_id, description)
VALUES (50, 0, 0, 1, 1);
INSERT INTO transaction_leg (monetary_amount, physical_energy, embodied_energy,
account_id, transaction_id, description)
VALUES (-50, 0, 0, 8, 1);
```

4.6.2 Example Transaction 2

```
Transaction and Transaction Legs for the Capital Expenditure

INSERT INTO transaction (transaction_id, description)
VALUES (1, 'Acquire Facilities');
INSERT INTO transaction_leg (monetary_amount, physical_energy, embodied_energy,
account_id, transaction_id, description)
VALUES (-10, 0, 0, 1, 1);
INSERT INTO transaction_leg (monetary_amount, physical_energy, embodied_energy,
account_id, transaction_id, description)
VALUES (10, 0, 100, 2, 1);
INSERT INTO transaction_leg (monetary_amount, physical_energy, embodied_energy,
account_id, transaction_id, description)
VALUES (0, 0, -100, 8, 1);
```

4.7 Accessing The Complete Implementation

The above snippets are meant to provide an illustration and explanation of the SQL code that is required to create and populate an SQL database that implements a simple version of the integrated energy accounting principles we discussed in this paper. The complete SQL code available at the (energyLedger repository) has a longer list of transaction examples, replicating the discussion in Section 3. The SQL implementation script can be executed in one step, subject to an existing installation of a Postgres database.

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