How to Random Walk on Supply and Use Tables (and why bother?)

Philippos Papadopoulos
Open Risk

16th Input-Output Workshop
March 27th 2025
Groningen, Netherlands

Motivation: Tools for Sustainable Portfolio Management

- Start with All of Finance (Includes Unsustainable Finance)
  - Focus: Sustainable Finance (Nebulous Term)
    - Focus more: Sustainable Portfolio Management (Practical Task within Sustainable Finance)
      - Focus further: Attribution of Environmental Impact to existing Portfolio (Present Tense, similar to Valuing a Portfolio, Accounting and Reporting)
        - EEIO helps specifically with attribution of *indirect* GHG Emissions (Scope 3)
      - Allocation of Resources (Future Tense, similar to Planning, Policies, Limit Frameworks and other management actions)

- EEIO embodies the holistic big picture, removes blind spots, leakages, etc.
- Potentially helps with accounting for other externalities
- It is used in combination with other approaches (LCA)
- In the PACTA methodology for banks it is considered a **fall-back option (last resort)**

Motivation: Adapt EEIO ideas for use in our context

Linking aggregate (macro) views to actual financial, insurance, procurement etc. portfolios and establishing relevance for the associated decision making processes is not trivial. Challenges along:

- The *Temporal Dimension*:
  - Timely updates / nowcasting etc.
- The *Granularity Dimension*:
  - Product resolution: More detailed product-sector structures?
  - Entity resolution: inhomogeneity within sectors, conglomerates etc.
- The *Fidelity Dimension*:
  - Revenue Flows versus Balance Sheets (Stocks)
  - The overall recognition and treatment of various economic actors

Motivation: There aren't many good alternatives!

Classic Portfolio Management uses Market Data to establish an implicit "economic network" structure

- Historical prices of traded capital instruments of entities (equity shares, bonds etc.)
- Econometric approach: Temporal *correlations* as evidence of affinity, association, dependency?
- Pros: timely, granular, directly relevant from financial perspective...
- Cons:
  - Economic reality obscured by market behaviors (bubbles, illiquidity)
  - In crises all correlations go to one
  - Very incomplete (mostly large corporates, no households etc.)
  - Environmental Impact data not a natural fit

Background: Input-Output Data and Graph Theory

Qualitative Input Output Analysis

- Long established concept (Solow 1952)
- Textbook application (Section 14.3 of Miller, Blair)
- Apply a quantization algorithm to convert the numerical values of an IO matrix into binary (1/0)
  - Adjacency Matrix of Directed Graph
  - Sectors → Vertices
  - Arcs → Economic Relations
- Less in more: reveal paths of influence and underlying structure

	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{10}	s_{11}
s_1		1		1	1	1	1				
s_2	1		1	1							
s_3				1							
s_4					1						
s_5	1		1	1							
s_6							1			1	
s_7								1	1		
s_8							1		1		
s_9								1			
s_{10}											1
s_{11}										1	

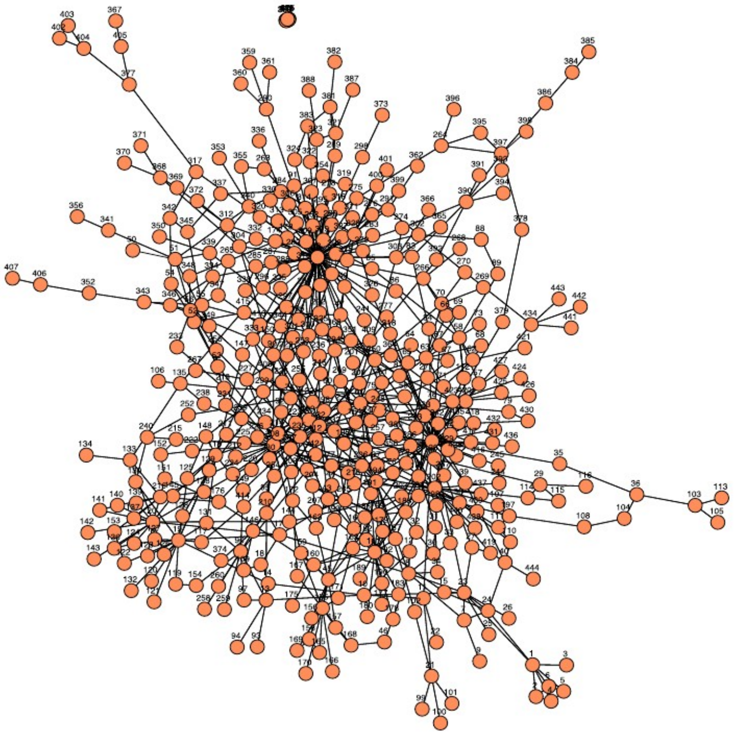
Holub, Schnabl 1985

Qualitative input-output analysis and structural information

Background: Input-Output Data and Network Theory

Embedded in larger Economic and Financial Network Models

- Study economic phenomena using network concepts
- E.g., *The Network Origins of Aggregate Fluctuations* (Acemoglou et al. 2012)
- Popular theme *after* the Financial Crisis
- E.g., *Recent advances in modeling systemic risk using network analysis* (European Central Bank, 2010)
- Use graph theory concepts to identify concentration, weak links, contagion etc.
- Specialized databases (e.g asset/liability links), available to regulators but not generally available

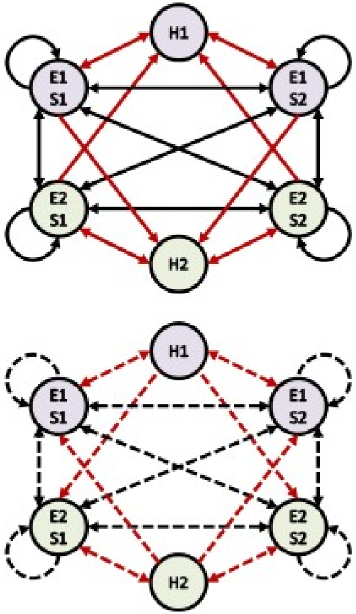


Intersectoral network corresponding to the U.S. input–output matrix in 1997.

Background: Symmetric IO and Markov Chains

Probabilistic interpretation of IO data

- Convert IO transaction data into probabilities
- Row / column stochastic options
- The Transition Matrix (Markov Chain) becomes the central object of interest
- Defines a discrete Random Walk from Sector node to Sector node
- Enables using the probability toolkit to study IO structures



Random Walks on the World Input -Output Network, Piccardi et al. 2018

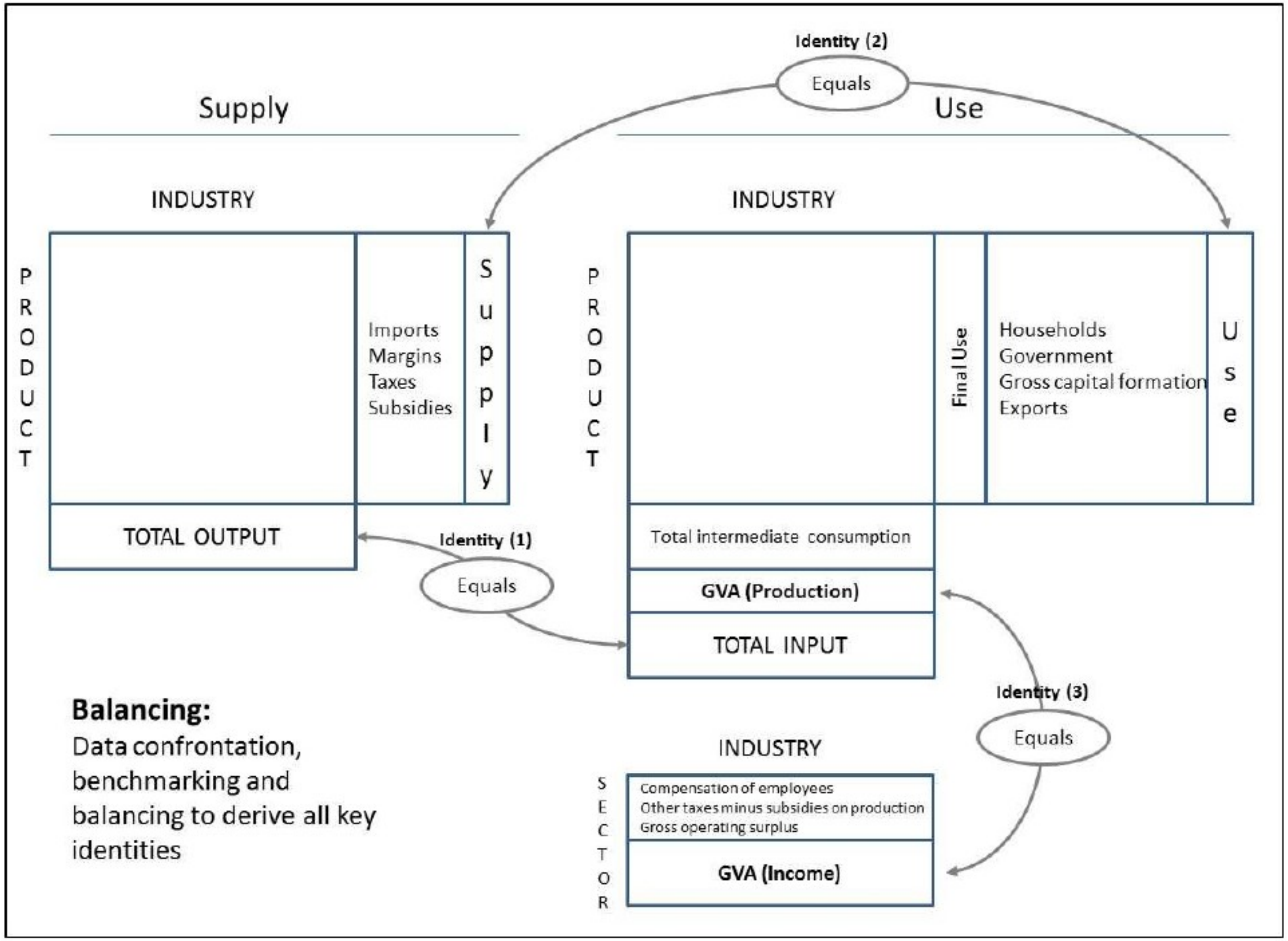
Leontief Meets Markov: Sectoral Vulnerabilities Through Circular Connectivity, Wirkierman et al. 2021

Organize a general SUT as a Graph with a Markov Chain

“General” here means: No requirement that Product ↔ Industry

No models A, B, C, D etc.

E.g., FIGARO-E3 database encompasses data for 213 products, 176 industries across 45 countries



Organize a general SUT as a Graph with a Markov Chain

The “Roadmap”

- Map the SUT data into a *graph data structure*
- Apply consistently to both Closed and Open systems (a conceptual alignment with Social Accounting Matrices)
- Incorporate extensions such as diverse Environmental Impacts
- Define associated *markov chain*(s)
- Compute useful things:
  - Tools from graph theory and from probability theory
  - New types of calculations?
- Write an *open source library* to facilitate all the above (LEONTIEF, in progress)
- Apply to actual SUT data (FIGARO-E3, in progress)

More details in a white paper

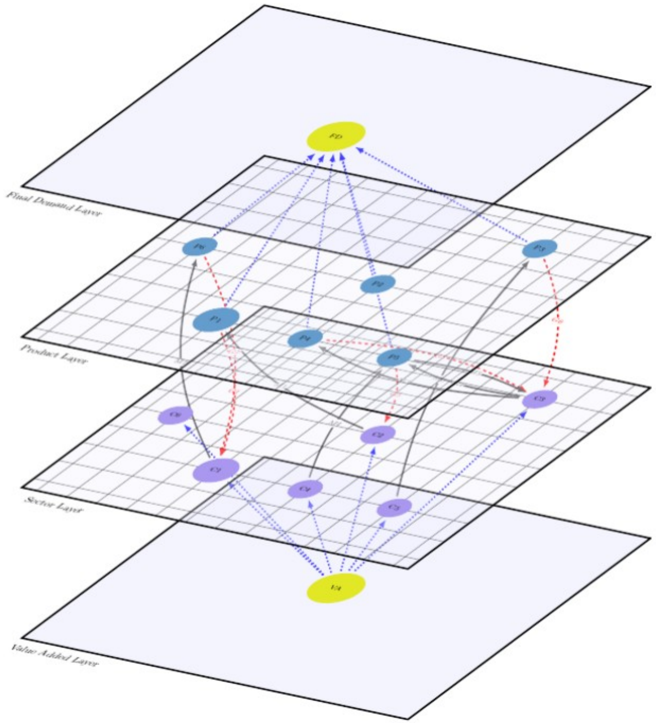
Open Risk White Paper

Follow the Money:
Random Walks on Supply and Use Graphs

Authors: Philippos Papadopoulos

December 16, 2024

Available Here



The Weighted Adjacency Matrix W of the SUT graph

$$W_{kl} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & U_{11} & U_{12} & \cdots & U_{1,n-1} & y_1 \\ 0 & 0 & \cdots & 0 & 0 & U_{21} & U_{22} & \cdots & U_{2,n-1} & y_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & U_{m-1,1} & U_{m-1,2} & \cdots & U_{m-1,n-1} & y_{m-1} \\ 0 & 0 & \cdots & 0 & 0 & v_1 & v_2 & \cdots & v_{n-1} & 0 \\ V_{11} & V_{12} & \cdots & V_{1,m-1} & 0 & 0 & 0 & \cdots & 0 & 0 \\ V_{21} & V_{22} & \cdots & V_{2,m-1} & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ V_{n-1,1} & V_{n-1,2} & \cdots & V_{n-1,m-1} & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

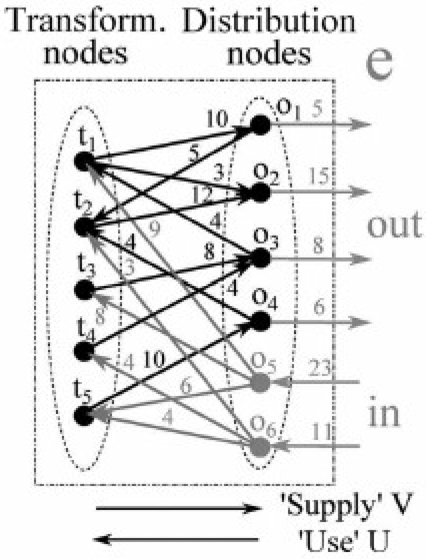
A characteristic Block Structure

Some related ideas from the literature

$$\begin{bmatrix} 0 & \mathbf{U} \\ \mathbf{V} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{e}_c \\ \mathbf{e}_i \end{bmatrix} + \begin{bmatrix} \mathbf{y}_c \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{q} \\ \mathbf{g} \end{bmatrix}$$

"Block formulation" helps to elucidate technology assumptions towards Symmetric IO

A note on the use of supply-use tables in impact analyses, Manfred Lenzen and José M. Rueda-Cantuche, 2012



Output, from transformation
Input, to distribution

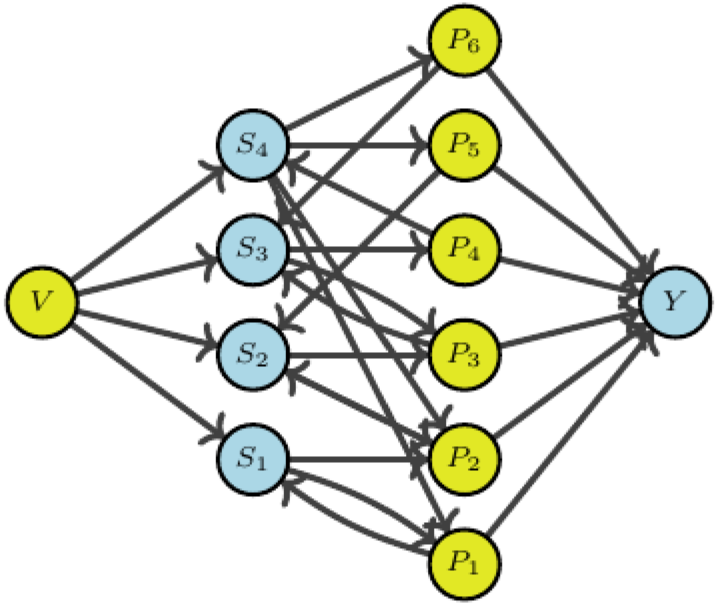
	t_1	t_2	t_3	t_4	t_5	e	q
o_1	10						10
o_2	3	12					15
o_3			8	4			12
o_4					10		10
o_5						23	23
o_6						11	11
g	13	12	8	4	10		

Generalized supply table

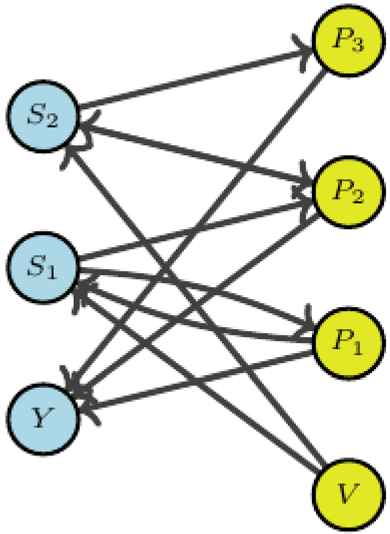
A General System Structure and Accounting Framework for Socioeconomic Metabolism, Pauliuk et al. 2015

SUT Systems are Bipartite Graphs

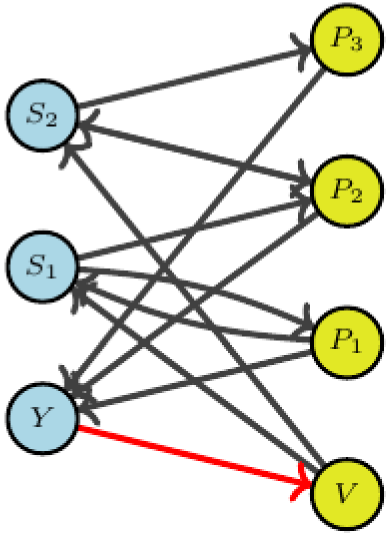
- Natural split into **Sector** and **Product** nodes
- Products can be *many more* than Sectors
- Sectors trade with other sectors *only* via Product Markets (this is the bipartite property)
- Two types of graph edges:
  - S → P (from Supply Table)
  - P → S (from Use Table)
- Value Added and Final Demand nodes are naturally mapped into P and S respectively
- Closed versus Open representations reflected in the connectivity of the nodes (absence of sinks and sources)



SUT Systems are Bipartite Graphs



Open SUT System in bipartite form. The Y node has no outgoing edges while the V node has no ingoing edges.



Closing the SUT System by adding an edge between Y and V. Only possible for a balanced system.

Upstream Transitions Matrix (Column Stochastic)

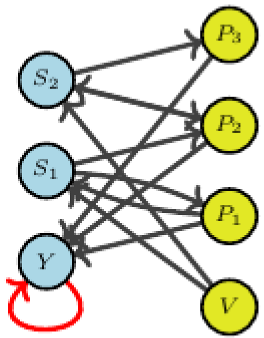
$$Q^{in}_{kl} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & u_{11} & u_{12} & \cdots & u_{1n} & \tilde{y}_1 \\ 0 & 0 & \cdots & 0 & 0 & u_{21} & u_{22} & \cdots & u_{2n} & \tilde{y}_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & u_{m1} & u_{m2} & \cdots & u_{mn} & \tilde{y}_m \\ 0 & 0 & \cdots & 0 & 1 & \tilde{v}_1 & \tilde{v}_2 & \cdots & \tilde{v}_n & 0 \\ v_{11} & v_{12} & \cdots & v_{1m} & 0 & 0 & 0 & \cdots & 0 & 0 \\ v_{21} & v_{22} & \cdots & v_{2m} & 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ v_{n1} & v_{n2} & \cdots & v_{nm} & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$Q^{out}_{kl} = \frac{W_{kl}}{z_l}$$

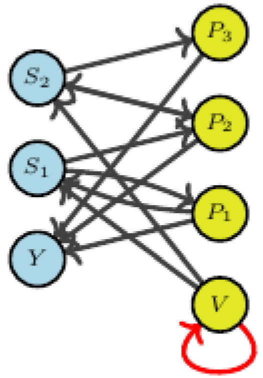
$$Q^{in}_{kl} = \frac{W_{kl}}{z_k}$$

Z is row/column sums of W

Various ways to construct absorbing chains



Absorbing Chain at Final Demand (Sink).
Downstream flow reaching Y will stay there.



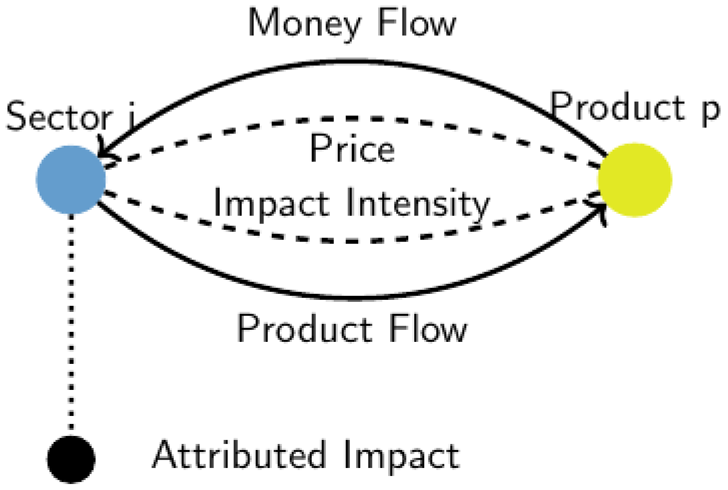
Absorbing Chain at Value Added (Source).
Upstream flow reach V will stay there.

Environmental Impact expressed via Graph Elements

In EEIO impacts are additional “rows”.

Where do environmental impacts “live” on the SUT graph?

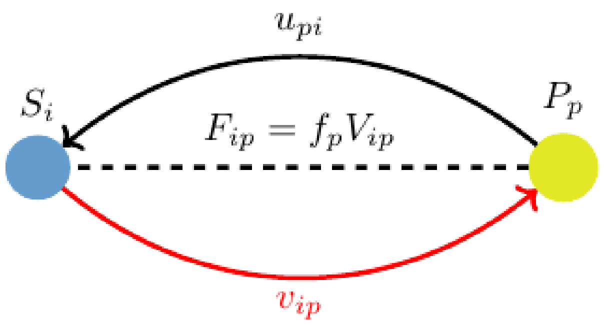
- Impact comes from production by firms but this is not modeled explicitly in SUT data. It is implicit in the supply of products to markets
- We thus associate impact with the product flow from the producing Sector to Product Market
- In principle impact is a **matrix** object (a sector and product specific intensity)
- Removing the sectoral dependency might be a reasonable simplification



The Follow the Money Algorithm

The high-level algorithm (Open System) for cumulating impact:

- Start with any non-absorbing node X(0)
- Follow the graph consistently in one direction using the appropriate transition matrix Q to compute jump probabilities to other nodes
- Every time (t) you cross an *emitting edge* (red type), add-up the associated impact F(t)
- Stop when reaching an absorbing node (value added products or final demand sectors)
- Algorithm can be implemented:
  - Literaly (as a *random walk simulation*)
  - Calculate *expectations* (linear framework)



X(0)=s0, X(1)=p1, X(2)=s1, X(3)=p2, etc.
Is the random walk from sector to product to sector etc.

Impact as Expectation of Random Walk on SUT

$$Q^{in}_{lk} = \Pr(X(t) = k \mid X(t-1) = l)$$

Q → Transition probabilities

P(t) -> probability distribution over nodes at step t

$$P(t) = Q^{in} P(t-1)$$

$$\begin{aligned}
\mathbb{E}(F(t)) &= \sum_k \sum_l \mathbb{E}(F(t)|X(t) = k, X(t-1) = l) \\
&= \sum_i \sum_p \mathbb{E}(F(t)|X(t) = p, X(t-1) = i) \\
&= \sum_i \sum_p f_{ip}\, P(X(t) = p, X(t-1) = i) \\
&= \sum_i \sum_p f_{ip}\, P(X(t) = p\,|X(t-1) = i) P(X(t-1) = i) \\
&= \sum_i \sum_p f_{ip} Q^{in}_{ip} P_p(t-1) \\
&= f \circ Q \cdot P(t-1)
\end{aligned}$$

The evolution of P(t) follows a Kolmogorov type equation (matrix – vector multiplication)

$$\mathbb{E}(C(t)) = \sum_t \mathbb{E}(F(t)) = \sum_t f \circ Q^t \cdot P(0)$$

Example Calculation (Replicating Kitzes 2013)

	Ag	Ma	FD	TO
Ag	8	5	3	16
Ma	4	2	6	12
VA	4	5		
TI	16	12		

$$Q_{kl} = \left[\begin{array}{c|cccccc} & Ag_p & Ma_p & VA & Ag_s & Ma_s & FD \\ \hline Ag_p & 0 & 0 & 0 & 0.50 & 0.42 & 0.33 \\ Ma_p & 0 & 0 & 0 & 0.25 & 0.17 & 0.67 \\ VA & 0 & 0 & 1 & 0.25 & 0.42 & 0 \\ Ag_s & 1 & 0 & 0 & 0 & 0 & 0 \\ Ma_s & 0 & 1 & 0 & 0 & 0 & 0 \\ FD & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

A “trivial” reversion from an IO system back to a SUT system:
All nodes split into pairs (P,S) along with a diagonal supply matrix

Example Calculation

Initial Distribution P(0) (X(0)=3)

P(0) =	[0	0	0	1	0	0]
P(1) =	[0.5	0.25	0.25	0	0	0]
P(2) =	[0	0	0.25	0.5	0.25	0]
P(3) =	[0.35	0.17	0.48	0	0	0]
P(4) =	[0	0	0.48	0.35	0.17	0]
P(5) =	[0.25	0.12	0.64	0	0	0]
P(6) =	[0	0	0.64	0.25	0.12	0]
P(7) =	[0.17	0.08	0.75	0	0	0]
P(8) =	[0	0	0.75	0.17	0.08	0]
P(9) =	[0.12	0.05	0.82	0	0	0]
P(10) =	[0	0	0.82	0.12	0.06	0]
P(11) =	[0.08	0.04	0.88	0	0	0]

K-th round impact

I(1) = 0.33
I(2) = 0
I(3) = 0.23
I(4) = 0
I(5) = 0.16
I(6) = 0
I(7) = 0.11
I(8) = 0
I(9) = 0.078
I(10) = 0
I(11) = 0.055

Convergence to the absorbing state (VA=2)

Environmental Impact “Variability”

- The computed first-order moments (impact expectations) correspond to the standard EEIO results
- Higher moments *postulate* a cloud of *combinatorial uncertainty* around the expectation
- NB: This is **not** data-quality related uncertainty!
- This variability reflects the network structure (it is lower for diversified network)
- The implicit assumption is that the Q’s are invariant in time. This is somewhat analogous to the assumption of Leontief / Ghosh model (A matrix)

$$\mathbb{V}(F(t)) = f^2 \circ Q \cdot P(t-1) - (f \circ Q \cdot P(t-1))^2$$
$$= f^2 \circ Q^t \cdot P(0) - (f \circ Q^t \cdot P(0))^2$$

$$F(t) \approx \mathbb{E}(F(t)) \pm \sqrt{\mathbb{V}(F(t))}$$

E.g., F(1) = 0.26 +/- 0.22

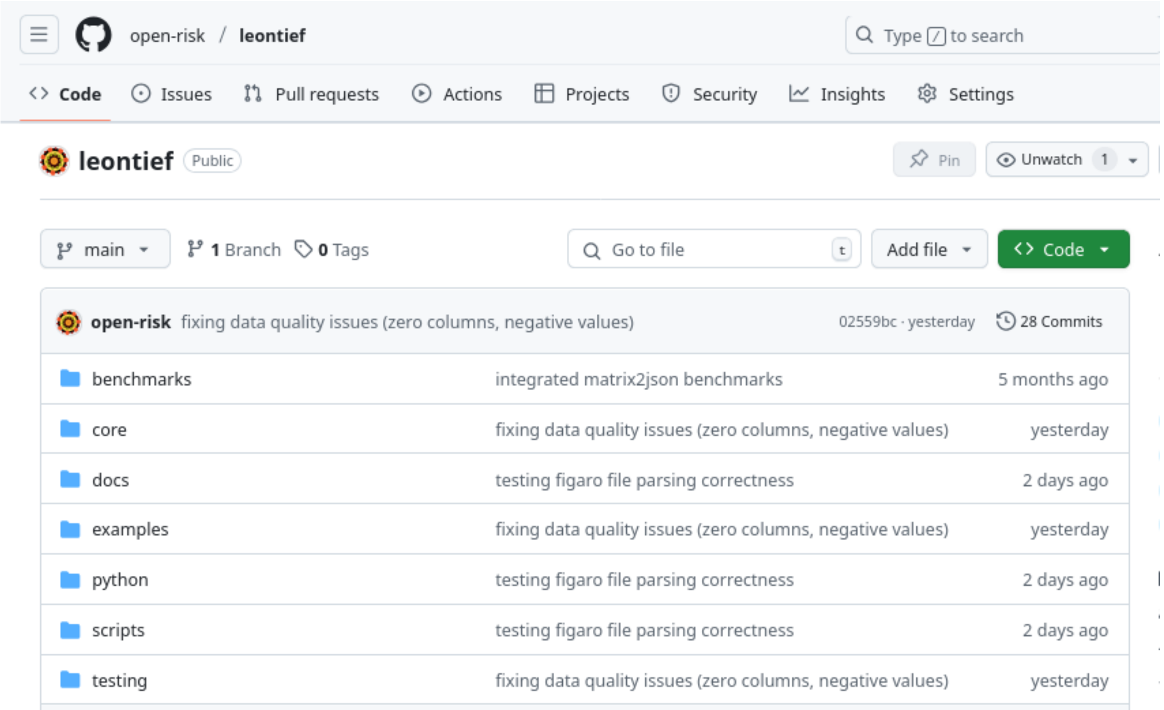
Conclusions

- Supply and Use tables can be mapped naturally into Bipartite Graph Networks (in various Open and Closed economy configurations)
- The EEIO extensions become additional graph elements
- It is possible to define Markov Chains and random walks (upstream or downstream) on these SUT graphs
- It is possible to compute both standard and new environmental impact metrics using the probabilistic interpretation

Open Source Tools

Leontief is a new open source toolkit to work with SUT graphs

- Striving for a balance between usability and performance
- Using the Eigen C++ linear algebra library (used by many Machine Learning / Data Science Projects)



Further Resources

Open Risk Academy

- Elearning environment
- Focus on technical / computational aspects
- Using open source software tools for learning
- Interactivities

Risk Management

- Risk Management Motivation
- The Shortest Possible Course on Risk Management
- Open Risk Crossword Puzzles
- The Periodic Table of Risk Elements
- Risk Management Questions and Answers

Sustainable Finance

- Input-Output Model Interactivities
- Crash Course on Input-Output Model Mathematics
- Introduction to Input-Output Models using Python
- Input-Output Models as Graph Networks
- The Climate Dictionary Quiz

THANK YOU!